Intermediate Microeconomic Theory
ECN 100B (Section A), Fall 2019

Professor Brendan Price

Final Exam

Name: ____________________________

ID number: ______________

- Write your answers on the exam itself, using only the space provided for each question.
  - If you run out of space for a question, write "see extra space" in the space provided for that question, then finish your answer in the extra graded space at the end of the exam. Be sure to write the question #. You may lose credit if we can't tell which question you're answering.
  - We've also included ungraded scrap space for pure scrap work. Answers written in this ungraded space will not be graded under any circumstances.
- Show your work on every question that requires a calculation. We will award partial credit as appropriate. Correct results without adequate work will receive little or no credit.
- Simplify all mathematical expressions as much as possible.
- There are six pages with questions (pages numbered 2, 3, 4, 5, 6, 7). After the exam starts, make sure that you have all of the pages and that your exam booklet is stapled properly. If there is a problem with your exam, we will give you a new copy.
- The exam is graded out of 60 points. Each question is worth the indicated number of points.
- You will have 120 minutes. You must drop your pen/pencil immediately when time is up. If you keep writing after time is called, we will deduct points.
- As a reminder: UC Davis has a strict code of Academic Conduct. Any violations, including copying or attempting to copy from another student, will result in a score of 0.
- Good luck!

Do not turn this page until I tell you to start.

1. **Up in the Air (4 pt.)**

   Demand for Uber rides to the airport is $p(Q) = 20 - Q$. Suppose Uber is a uniform-pricing monopolist.

   a. (2 pt.) Suppose Uber's costs are $C(Q) = 4Q + Q^2$. Write Uber's profits as a function of Q. Then find Uber's profit-maximizing choice Q_m.

      Uber's profits are $\pi(Q) = (20 - Q)Q - 4Q - Q^2$. FOC: $20 - 2Q - 4 - 2Q = 0 \implies Q_m = 4$.

   b. (2 pt.) Now suppose Uber's costs are $C(Q) = 24Q$. Find Uber's profit-maximizing choice Q_m. What is the socially optimal quantity, Q_s?

      Uber's MC always exceeds its MR, so its optimal choice is $Q_m = 0$. (If you take the FOC and solve, you'll get a negative value $Q = -2$, indicating a corner solution.) The socially optimal quantity is $Q_s = 0$ as well: the benefits to society from the first transaction are also smaller than the cost to society.

2. **Wall Street (6 pt.)**

   Australis and Borealis are equally promising companies: each share of either Australis or Borealis stock is worth \$6 with probability $\frac{1}{2}$ and \$0 with probability $\frac{1}{2}$. The stock prices are uncorrelated.

   a. (2 pt.) Find the expected value and variance of a portfolio consisting of <u>two</u> shares of Australis.

      Expected value: $\mathbb{E}(2X_A) = \frac{1}{2}(12) + \frac{1}{2}(0) = 6$. Variance: $\text{Var}(2X_A) = \frac{1}{2}(12-6)^2 + \frac{1}{2}(0-6)^2 = 36$.

   b. (3 pt.) Find the expected value and variance of a portfolio consisting of one share of Australis and one share of Borealis. If she can buy both portfolios at the same price, which portfolio—the first one (from part a) or the second one (part b)—would a risk-averse investor prefer to own?

      Expected value: $\mathbb{E}(X_A + X_B) = \frac{1}{4}(12) + \frac{1}{2}(6) + \frac{1}{4}(0) = 6$. Variance: $\text{Var}(X_A + X_B) = \frac{1}{4}(12 - 6)^2 + \frac{1}{2}(6 - 6)^2 + \frac{1}{4}(0 - 6)^2 = 18$. A risk-averse investor would prefer the portfolio in part b.

   c. (1 pt.) Suppose you own 50 shares of Australis and 50 shares of Borealis. If you are risk-averse, would you prefer that the price of Australis stock be *perfectly positively correlated* with the price of Borealis stock, *perfectly negatively correlated*, or *uncorrelated*? (No need to explain.)

      You would like Australis and Borealis to be perfectly negatively correlated, since a negative correlation reduces (in this case, eliminates) the variance of your diversified portfolio—whenever Australis is doing badly, Borealis is doing well, which stabilizes the value of your asset holdings.

3. **Quidditch (4 pt. total)**

This year's Quidditch final is Ravenclaw vs. Hufflepuff (brought to you by Nimbus Broomsticks).

		Keeper		
		Block	Scoot	Blink
Chaser	Lob	5, 2	8, 0	9, 1
	Slam	2, −7	8, 1	3, 0
	Slip	0, 6	−3, 5	1, 1

a. (2 pt.) Suppose this is a static game. Circle all payoffs corresponding to a player's best response, then list all pure strategy Nash equilibria (or write "none" if there aren't any). Be sure to write the *strategies*, not payoffs.

The PSNEs are (Lob, Block) and (Slam, Scoot).

b. (1 pt.) Again treating this as a static game, identify all strictly dominated strategies (or write "none" if none).

Slip and Blink are both strictly dominated.

c. (1 pt.) Now Chaser moves first. In the subgame-perfect Nash equilibrium, which action does Chaser choose?

Chaser will choose Slam.

4. **Attitude problems (6 pt.)**

For each situation, indicate whether the agent wants to buy the good being described by choosing one of these four options: "definitely buys", "definitely doesn't buy", "indifferent", or "not enough information". (Each option may be used once, more than once, or not at all.) If the question refers to a utility function $u(w)$, assume that $w > 0$. You don't have to show your work here.

a. (1 pt.) A shot of espresso costs \$3. My reservation price is \$2.
   Definitely doesn't buy.

b. (1 pt.) Someone with utility function $u(w) = w^2$ is deciding whether to buy a lottery ticket. The transaction is a fair bet.
   Definitely buys.

c. (1 pt.) A risk-averse agent is offered pet insurance at a price above the actuarially fair price.
   Not enough information.

d. (1 pt.) Someone with utility function $u(w) = 10w + 3$ is deciding whether to buy car insurance. The insurance premium is \$30, and the expected claim is \$30.
   Indifferent.

e. (1 pt.) Someone with utility function $u(w) = 5\sqrt{w}$ is deciding whether to buy a lottery ticket. The ticket costs \$10. Its expected value is \$8, and it has positive variance.
   Definitely doesn't buy.

f. (1 pt.) Someone with a linear utility function is deciding whether to buy a stock portfolio. The portfolio costs \$20 to buy. Its expected value is \$25, and it has positive variance.
   Definitely buy.

5. **Swipe (4 pt.)**

Tolu starts with $w = 400$ in total wealth, consisting of \$100 cash plus a \$300 laptop, but there is a 10% chance that her laptop gets stolen (leaving her with just $w = 100$). Her utility is $u(w) = \sqrt{w}$.

a. (2 pt.) Compute Tolu's expected utility given this risk. Then compute her certainty equivalent.

Expected utility: $\frac{1}{10}\sqrt{100} + \frac{9}{10}\sqrt{400} = 19$. Certainty equivalent: $\sqrt{CE} = 19 \implies CE = 361$.

b. (2 pt.) A startup called Swipe provides full insurance against the risk of laptop theft. What is the actuarially fair insurance premium? How much would Tolu be willing to pay for full insurance?

The actuarially fair premium equals the expected claim: $\frac{1}{10} \times 300 = 30$. Since Tolu's CE is 361, she is indifferent between getting \$361 for sure and playing the "lottery" of possibly getting her laptop stolen. Since she'd be willing to give up \$39 to avoid this risk, her WTP is \$39.

6. **On the Road Again (6 pt.)**

Freewheeler Bicycle Center offers (excellent!) bike repairs using labor as its only input, with production function $q(L) = A\sqrt{L}$, where A is a constant. It's a price-taker ($p = 5$) and a wage-taker ($w = 10$).

a. (2 pt.) Suppose that $A = 12$. Compute the marginal physical product of labor (MPPL) as a function of L. Compute the marginal revenue product of labor (MRPL) as a function of L.

MPPL: $q'(L) = \frac{6}{\sqrt{L}}$. MRPL: $pq'(L) = \frac{30}{L}$.

b. (2 pt.) Keep assuming $A = 12$. Write profit as a function of L, then find the optimal choice L^*.

$\max_L \ \pi = pq(L) - wL \implies \max_L \ \pi = 60\sqrt{L} - 10L$. FOC: $\frac{30}{\sqrt{L}} = 10 \implies L^* = 9$.

In class, we assumed firms can freely adjust L in response to "short-run" changes in market conditions. In practice, hiring new workers takes time, and labor (like capital) may be fixed in the short run.

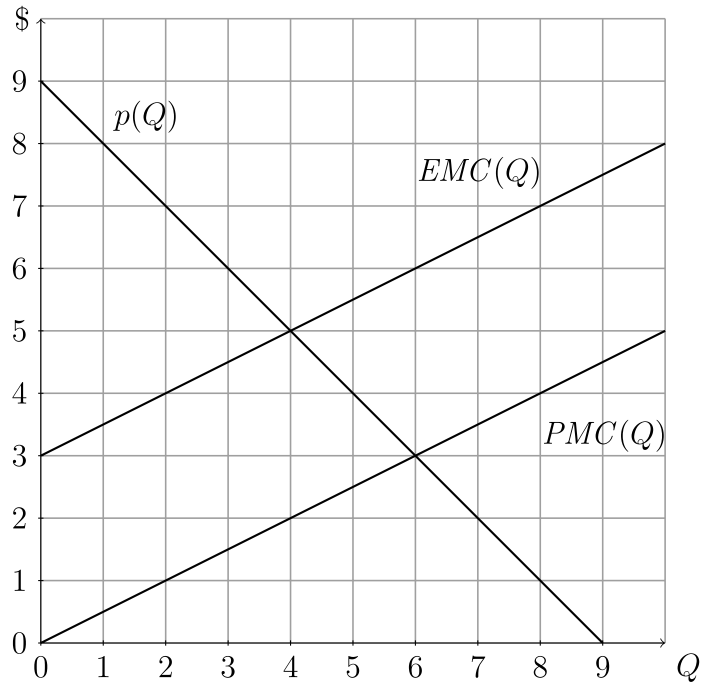
Suppose that Freewheeler's workers learn new repair techniques, causing A to increase above 12. (Assume that p and w are unaffected by this change. Freewheeler still does not use capital.)

c. (1 pt.) In the short run, suppose L stays at the value L^* you found in part b. Relative to their values when $A = 12$, the MPPL will rise (rise/fall/not change) and the MRPL will rise (rise/fall/not change).

d. (1 pt.) In the long run, Freewheeler will adjust its choice of L to a new optimum. Relative to their values when $A = 12$, the MPPL will not change (rise/fall/not change) and the MRPL will not change (rise/fall/not change).

7. **Negative externalities (10 pt.)**

Answer each question using the graph below. (You do not need to show your work here.)
$p(Q)$: demand. $PMC(Q)$: private marginal cost. $EMC(Q)$: external marginal cost.



Hint: the cost functions plotted above are $PMC(Q) = \frac{1}{2}Q$ and $EMC(Q) = \frac{1}{2}Q + 3$.

a. (3 pt.) In the competitive equilibrium,

- the quantity sold (Q_c) equals $\underline{6}$.
- the price equals $\underline{3}$.
- the producer surplus equals $\underline{9}$.

b. (2 pt.) Under perfect price discrimination,

- the quantity sold (Q_{ppd}) equals $\underline{6}$.
- the profit is $\underline{27}$.

c. (3 pt.) At the social optimum,

- the quantity (Q_s) equals $\underline{3}$.
- the social marginal cost equals $\underline{6}$.
- the deadweight loss equals $\underline{0}$.

d. (2 pt.) We can achieve the social optimum using a corrective $\underset{\text{(tax or subsidy)}}{\underline{\text{tax}}}$ equal to $\underline{4.5}$.

8. **Broken robots (5 pt.)**

You run a factory with production function $q(L, K) = 5L + 4K$, where L is humans and K is robots. Throughout this problem, your job is to produce $q = 40$ units of output at the lowest possible cost.

a. (2 pt.) Let $w = 4$ and $r = 3$. How much would it cost to produce $q = 40$ using (only) humans? How much would it cost to use (only) robots? Find the cheapest combination of L^* and K^*.

You can get the job done using 8 humans (which costs \$32), 10 robots (which costs \$30), or some combination of the two. It's cheapest to use robots, so $L^* = 0$ and $K^* = 10$.

b. (2 pt.) New technologies often break. Suppose $w = 4$ as before, but now r is a random variable: after you choose L and K, there's a 10% chance that any robots you rented require expensive repairs, so that $r = 6$. (Otherwise, the robots work fine and $r = 3$.) Calculate the expected cost of using robots. If your goal is to minimize the expected cost, will you use humans or robots?

Expected cost $= 0.1 \cdot (10 \times 6) + 0.9 \cdot (10 \times 3) = 33$. You should pay \$32 to hire the humans, since that's cheaper on average than hiring (and sometimes fixing) robots.

c. (1 pt.) Now suppose that—before you choose L and K—you can pay a mechanic to determine whether the robots will break. What is the most you'd be willing to pay the mechanic for this information (i.e., to learn the true value of r)? (Note: the answer is not an integer.)

If you don't pay the mechanic, you'll hire humans at a cost of \$32. If you pay the mechanic an amount x, then there's a 10% chance you'll get bad news (broken robots), in which case you'll hire humans and pay a total cost $32 + x$. But there's a 90% chance that you'll get good news (working robots), in which case you'll rent robots and pay $30 + x$. You're indifferent if

$$32 = 0.1 \cdot (32 + x) + 0.9 \cdot (30 + x) \implies x^* = 1.8$$

so your WTP for this information is \$1.80.

9. **True or false (5 pt. total)**

Indicate whether each of the following statements is true or false. (You don't have to explain why.)

a. (1 pt.) In a perfectly competitive market, the price markup equals 0%. True

b. (1 pt.) If a firm is a price-taker, then it faces downward-sloping demand ($p'(Q) < 0$). False

c. (1 pt.) Engaging in group price discrimination requires more information than engaging in perfect price discrimination does. False

d. (1 pt.) If a uniform-pricing monopolist is facing inelastic demand, then increasing its price will increase its profits. True

e. (1 pt.) In a Leontief production function, labor and capital are perfect substitutes. False

10. **Time to Move On (10 pt.)**

Abdul and Blake are about to graduate, but first they have to clean their apartment. (The messier they leave it, the louder the voicemail they get from their landlord, and the guiltier they feel.) Let

$$Q = q_A + q_B$$

denote total time spent cleaning, where q_A and q_B are time spent by Abdul and Blake, respectively. Each roommate receives private marginal benefits given by

$$p_A(Q) = 10 - Q \qquad \text{and} \qquad p_B(Q) = 6 - Q$$

a. (1 pt.) If the building's laundry room is free to use and doesn't have enough washing machines, we might call it a "common good". A common good is rival and non-excludable.

b. (2 pt.) Express the social marginal benefit in terms of Q. (Remember: it may have two parts.)

The SMB curve is $p(Q) = \begin{cases} 16 - 2Q & \text{if } Q \leq 6 \\ 10 - Q & \text{if } Q > 6 \end{cases}$.

c. (2 pt.) Suppose the marginal cost is 20 for both people. Find the socially optimal quantity (Q^{soc}). Assuming this is a static game, find Abdul and Blake's Nash quantities (q_A^{Nash} and q_B^{Nash})?

Even at $Q = 0$, $SMC > SMB$, so the social optimum is $Q^{\text{soc}} = 0$. Each roommate's PMB is less than her PMC, so choosing zero is strictly dominant for both roommates: $q_A^{\text{Nash}} = q_B^{\text{Nash}} = 0$.

d. (3 pt.) Now suppose the marginal cost is 8 for both people. Find the socially optimal quantity (Q^{soc}). Assuming this is still a static game, find the Nash quantities (q_A^{Nash} and q_B^{Nash}).

The SMB intersects the SMC on the first part of the SMB curve, so the optimum solves $16 - 2Q = 8 \implies Q^{\text{soc}} = 4$. For Blake, the PMC always exceeds the PMB, so Blake chooses $q_B^* = 0$ (strictly dominant strategy). Abdul's best response against $q_B^* = 0$ is to solve $10 - q_A = 8 \implies q_A^* = 2$.

e. (2 pt.) Suppose Abdul's marginal cost is still constant at $MC_A = 8$, but Blake's is $MC_B(q_B) = q_B$. Furthermore, suppose Abdul moves first (he's leaving town early). Find Blake's best-response function, $q_B^* = BR_B(q_A)$. Then find the subgame-perfect Nash quantities q_A^{SPNE} and q_B^{SPNE}.

Given Abdul's choice q_A, Blake chooses q_B to set his PMB equal to his PMC:

$$6 - q_A - q_B = q_B \implies q_B^* = BR_B(q_A) = 3 - \frac{1}{2}q_A$$

The next part is tricky. If Abdul chooses $q_A = 0$, then Blake best-responds by choosing $q_B^* = 3$. In that case, Abdul's PMC is 8, but his PMB from a slight increase in Q is only 7, so Abdul has no incentive to clean and he chooses $q_A^* = 0$. (In fact, for each unit of cleaning Abdul provides, total cleaning only increases by half a unit because Blake's effort is partly crowded out. That makes cleaning even less attractive for Abdul.) Blake's best response is then $q_B^* = 3$.