Intermediate Microeconomic Theory
ECN 100B (Section A), Fall 2019

Professor Brendan Price

Midterm Exam #1

Name: ____________________________

ID number: _________

- Write your answers on the exam itself, using only the space provided for each question.
  - If you run out of space for a given question, write "see extra space" in the space provided for that question, then finish your answer on the extra graded pages. Make sure to write the problem number. You may lose credit if we can't tell which question you're answering.
  - We've also included ungraded scrap pages for pure scrap work. Answers written on these ungraded pages will not be graded under any circumstances.
- You must show your work on every question that requires a calculation. We will award partial credit as appropriate. Correct results without adequate work will receive little or no credit.
- Simplify all mathematical expressions as much as possible.
- The exam is graded out of 50 points. Each question is worth the indicated number of points.
- You will have 80 minutes. You must drop your pen/pencil immediately when time is up.
- As a reminder: UC Davis has a strict code of Academic Conduct. Any violations, including copying or attempting to copy from another student, will result in a score of 0.
- Good luck!

Do not turn this page until I tell you to start.

1. **True or false (10 points total)**

Indicate whether each of the following statements is true or false. Provide a brief explanation (1–3 sentences) justifying your answer.

a. (2 pts.) Allowing a firm to engage in price discrimination (if it chooses to do so) can sometimes reduce its profits.

False. If we allow a firm to engage in price discrimination, it can still charge everybody the same price if doing so is profit-maximizing. We are simply giving the firm additional options: this might increase their profits, or it might leave their profits unchanged, but it can never decrease their profits.

b. (2 pts.) In an optimization problem with one choice variable, the solution to the first-order condition is always the optimal choice.

False. There are two main reasons. First, there may be a corner solution, which occurs when the solution to the FOC does not belong to the choice set (e.g., firms cannot produce negative output). Second, the solution to the FOC might be a minimum when we're really looking for a maximum, or vice versa. Or there may be multiple solutions to the FOC, in which case some of them may be local optimums but not global optimums.

c. (2 pts.) Boeing produces aircraft using a combination of labor (L) and capital (K). If the cost of renting capital goes up ($r \uparrow$), Boeing might reduce its demand for labor ($L^* \downarrow$).

True. When $r \uparrow$, the substitution effect causes Boeing to substitute away from capital and towards labor (increasing L^*), but the scale effect causes it demand less of both labor and capital (decreasing L^*). If the scale effect is stronger, then Boeing will hire fewer workers.

d. (2 pts.) If a car dealership engages in perfect price discrimination, it sells a car to everyone who has positive willingness to pay.

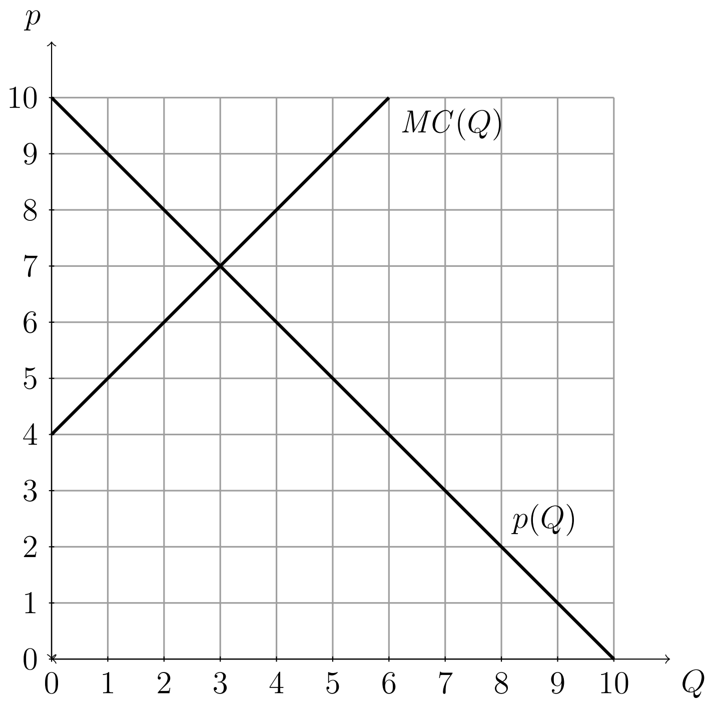
False. It only sells cars to people whose willingness to pay is at least as great as the marginal cost of selling them a car.

e. (2 pts.) Sometimes imposing a price cap increases total surplus.

True. For example, in a monopolized market, setting a price cap at the competitive level will get the monopolist to produce the competitive quantity, which is socially optimal.

2. **Graphical questions (10 points total)**

Fill in the blanks using the graph below. (You do not need to show your work here.)



a. Suppose that the market shown above is perfectly competitive.

   i. (2 pts.) The equilibrium price is 7. The producer surplus is 4.5.

   ii. (1 pt.) If producers have to pay a $2 tax for each unit sold, the DWL is 1.

b. Now suppose that the market shown above represents perfect price discrimination.

   i. (1 pt.) The firm sells to anyone with a reservation price between 7 and 10.

   ii. (1 pt.) The total revenue from the firm's sales is 25.5.

c. Now suppose that the market shown above represents a uniform-pricing monopoly.

   i. (2 pts.) The monopoly quantity is 2. The monopoly profit is 6.

   ii. (1 pt.) At the monopoly's optimal price, the markup equals 1/3.

   iii. (1 pt.) We can get the monopoly to produce the competitive quantity by setting a price *ceiling* (floor or ceiling) equal to 7.

   iv. (1 pt.) Total *revenue* is maximized at the point where $Q = 5$ units.

3. **Scraping by (10 points total)**

Sunil runs a pizzeria, which faces demand given by $p(Q) = 20 - \frac{1}{2}Q$. His variable costs are $VC(Q) = 10Q$. He has already paid a fixed cost $FC = 120$ to enter the market.

a. (3 pts.) Compute the elasticity of demand as a function of Q. For what value of Q are consumers least price-sensitive? For what value of Q is demand unit elastic?

The elasticity is given by $\varepsilon = 1 - \frac{40}{Q}$. Consumers are least price-sensitive when $\varepsilon = 0$, which occurs when $Q = 40$. Demand is unit elastic when $\varepsilon = -1$, i.e., when $Q = 20$.

b. (5 pts.) Suppose that Sunil is a uniform-pricing monopolist.

i. Write Sunil's profits as a function of Q. (Include the fixed cost.)

Profit is $\pi(Q) = (20 - \frac{1}{2})Q - 10Q - 120$, which simplifies to $\pi(Q) = 10Q - \frac{1}{2}Q^2 - 120$.

ii. Assuming he stays in business, what quantity (Q_m) and price (p_m) will he choose?

The FOC is $10 - Q = 0 \implies Q_m = 10 \implies p_m = 15$.

iii. If his fixed cost is (100%) recoverable, will he stay in business or exit?

Staying in the market yields profit $\pi = 15 \cdot 10 - 10 \cdot 10 - 120 = 50 - 120 = -70$. Since the cost is recoverable, leaving yields profits of 0. Since $0 > -70$, Sunil exits.

iv. If his fixed cost is (100%) sunk, will he stay in business or exit?

In this case, leaving yields profits of -120. It's better to stay in the market and cut his losses, getting profits of -70.

c. (2 pts.) Sunil takes ECN 100B, studies hard, and learns how to perfectly price discriminate.

i. Assuming he stays in business, how many pizzas will he sell (Q^*)?

He sells to anyone with $WTP \geq MC$: $p(Q^*) = MC \implies 20 - \frac{1}{2}Q^* = 10$, so $Q^* = 20$.

ii. If he can recover 50% of his fixed costs, will he stay in business or exit?

If he stays, his profit is the area below the demand curve and above the cost curve from $Q = 0$ to $Q = 20$, which is 100, minus the full fixed cost, so he gets $\pi = 100 - 120 = -20$. If he exits, his profit equals $-120 \times 50\% = -60$. So he should stay in business.

4. **Cheap talk (4 points total)**

In each of the following cases, find the cheapest combination of labor and capital needed to produce 1 unit of output. (L and K don't have to be integers: for example, L^* could equal $\frac{3}{2}$.) Also state whether labor and capital are perfect substitutes, perfect complements, or neither.

a. (2 pts.) $q(L, K) = \sqrt{LK}$, with $w = 2$, $r = 32$

We solve the cost-minimization problem

$$\min_{L,K} \ wL + rK \quad \text{subject to} \quad \sqrt{LK} = 1$$

Isolating K in the constraint gives $K = \frac{1}{L}$. Plugging this into the objective function:

$$\min_{L} \ wL + \frac{r}{L} = 2L + \frac{32}{L}$$

which has the FOC: $2 - \frac{32}{L^2} = 0 \implies L^* = 4 \implies K^* = \frac{1}{4}$. Labor and capital are neither perfect substitutes nor perfect complements.

b. (2 pts.) $q(L, K) = \min\{3L, 2K\}$, with $w = 4$, $r = 6$

Set $3L = 2K = 1$, yielding $L^* = \frac{1}{3}$ and $K^* = \frac{1}{2}$. (The answer doesn't depend on w or r.) Labor and capital are perfect complements.

5. **Not *another* Starbucks... (6 points total)**

Starbucks produces coffee according to the production function $q(L) = 8\sqrt{L}$. It's a price-taker in the product market, at price $p = 10$, and a wage-taker at wage $w = 10$.

a. (2 pts.) Compute the marginal physical product of labor in terms of L. Then compute the marginal revenue product of labor.

The MPPL is $q'(L) = \frac{4}{\sqrt{L}}$ and the MRPL is $pq'(L) = \frac{40}{\sqrt{L}}$.

b. (2 pts.) Find the profit-maximizing choice of labor L^*. Then compute Starbucks's profits.

We find L^* by setting MRPL equal to the wage: $\frac{40}{\sqrt{L}} = 10 \implies L^* = 16$. The profit is

$$\pi = pq(L) - wL = 80\sqrt{L} - 10L = 80 \cdot 4 - 10 \cdot 16 = 160$$

c. (2 pts.) Suppose that Starbucks is deciding whether to enter this market. If it enters, it must pay a fixed cost $FC = 40$. What is the smallest value of p for which Starbucks enters?

If Starbucks enters, it chooses L to maximize profits:

$$\max_{L} \ \pi = pq(L) - wL - FC = 8p\sqrt{L} - 10L - 40$$

The FOC is $\frac{4p}{\sqrt{L}} - 10 = 0 \implies L^*(p) = \frac{4}{25}p^2$. Plugging this choice back into the profit function gives us profits as a function of p:

$$\pi = 8p \cdot \left(\frac{2}{5}p\right) - 10 \cdot \frac{4}{25}p^2 - 40 = \frac{8}{5}p^2 - 40$$

Starbucks is indifferent about entering if profit equals zero, which happens when

$$\frac{8}{5}p^2 - 40 = 0 \implies p = 5$$

So Starbucks enters as long as $p \geq 5$.

6. **Tutor time (10 points total)**

Jo runs a tutoring agency that tutors students both in-person ("good 1") and online ("good 2").

- Demand for in-person tutoring is given by $p_1(Q_1) = 24 - 3Q_1$.
- Demand for online tutoring is perfectly elastic, given by $p_2 = 12$.
- Jo can provide any combination of the two goods at a cost $C(Q_1, Q_2) = (Q_1 + Q_2)^2$.
- Jo must charge all in-person customers the same price.

a. (3 pts.) Write Jo's profits as a function of Q_1 and Q_2. What is $MR_1(Q_1)$? What is $MR_2(Q_2)$?

Jo's profits are

$$\pi(Q_1, Q_2) = (24 - 3Q_1)Q_1 + 12Q_2 - (Q_1 + Q_2)^2$$

The marginal revenues are $MR_1(Q_1) = 24 - 6Q_1$ and $MR_2(Q_2) = 12$.

b. (3 pts.) Find Q_1^*, Q_2^*, and p_1^*.

The two FOCs are

$$24 - 6Q_1^* = 2(Q_1^* + Q_2^*)$$
$$12 = 2(Q_1^* + Q_2^*)$$

Solving this system of equations gives $Q_1^* = 2$, $Q_2^* = 4$, and $p_1^* = 18$.

c. (2 pts.) Suppose that p_2 increases. How will this affect Q_1^*, Q_2^*, and p_1^*? (Indicate whether each outcome increases, decreases, or stays the same. To receive credit, you must get all three comparative statics correct. You don't have to show your work on this part.)

$Q_1^* \downarrow$, $Q_2^* \uparrow$, and $p_1^* \uparrow$. Intuitively, an increase in p_2 increases MR_2, so Jo increases her choice Q_2^*. Suppose that Q_1^* stays the same. In that case, $Q_1^* + Q_2^*$ goes up, which implies that MC goes up. But $MR_1(Q_1^*)$ hasn't changed, which means that $MR_1(Q_1^*) < MC$—and that can't be optimal. So Jo compensates by reducing her output of good 1 until all of the FOCs are satisfied again. Reducing the supply of good 1 drives up its price, so that $p_1^* \uparrow$.

We can establish the same results mathematically by re-solving the original problem leaving p_2 unspecified. Assuming we still have interior solutions, doing so gives us the solutions

$$Q_1^*(p_2) = 4 - \frac{1}{6}p_2$$
$$Q_2^*(p_2) = \frac{2}{3}p - 4$$
$$p_1^*(p_2) = 12 + \frac{1}{2}p_2$$

If p_2 rises high enough, we get a corner solution in which $Q_1^* = 0$, which we can think of as corresponding to a price $p_1^* > 24$ (so that nobody wants to buy good 1). And Jo will produce a lot of good 2 (Q_2^* will definitely be greater than the value of 4 we found in part b). So the same comparative statics continue to hold if we imagine p_2 rising so high that we get a corner solution instead of an interior solution.

d. (2 pts.) Now suppose Jo has to charge in-person and online customers the same price p. Find all profit-maximizing prices p^*. (There may be only one, but there may be two.)

We have to consider two cases. First, Jo can choose $p > 12$, in which case $Q_2 = 0$ and she only sells to in-person customers. In this case, Jo solves the problem

$$\max_{Q_1} \pi = (24 - 3Q_1)Q_1 - Q_1^2$$

The FOC is $24 - 6Q_1 - 2Q_1 = 0 \implies Q_1^* = 3$ (and $Q_2^* = 0$). This corresponds to choosing the price $p^* = 15$, which yields profits $\pi = 15 \cdot 3 - 3^2 = 36$.

The second case is that Jo can choose $p = 12$, in which case she can sell as many units as she wants (she doesn't care whether they're in-person or online). In this case, she solves

$$\max_{Q} \pi = 12Q - Q^2 \implies 12 - 2Q = 0 \implies Q^* = 6 \implies \pi = 12 \cdot 6 - 6^2 = 36$$

So this approach also yields profits of 36. Since the prices $p^* = 15$ and $p^* = 12$ yield the same profits, they are both profit-maximizing.

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