Intermediate Microeconomic Theory
ECN 100B (Section A), Fall 2019

Professor Brendan Price

Midterm Exam #2

Name: ____________________________

ID number: __________________

- Write your answers on the exam itself, using only the space provided for each question.
  - If you run out of space for a given question, write "see extra space" in the space provided for that question, then finish your answer on the extra graded pages. Make sure to write the problem number. You may lose credit if we can't tell which question you're answering.
  - We've also included one ungraded scrap page for pure scrap work. Answers written on this ungraded page will not be graded under any circumstances.
- You must show your work on every question that requires a calculation. We will award partial credit as appropriate. Correct results without adequate work will receive little or no credit.
- Simplify all mathematical expressions as much as possible.
- The exam is graded out of 50 points. Each question is worth the indicated number of points.
- You will have 80 minutes. You must drop your pen/pencil immediately when time is up.
- As a reminder: UC Davis has a strict code of Academic Conduct. Any violations, including copying or attempting to copy from another student, will result in a score of 0.
- Good luck!

Do not turn this page until I tell you to start.

1. **Air show (8 pt. total)**

Amelia and Chuck play a static game. Answer each question based on the payoff matrix below.

		Chuck	
		Pitch	Roll
Amelia	Up	3, 2	(4), (6)
	Mid	(5), (2)	(4), 1
	Down	0, (14)	1, 12

a. (4 pt.) Circle every payoff that corresponds to a player's best response. Then identify all pure strategy Nash equilibria. (If there aren't any, write "none".)

Two PSNEs: (Up, Roll) and (Mid, Pitch).

b. (2 pt.) Amelia's best response against Pitch is __Mid__. Chuck's best response against Down is __Pitch__.

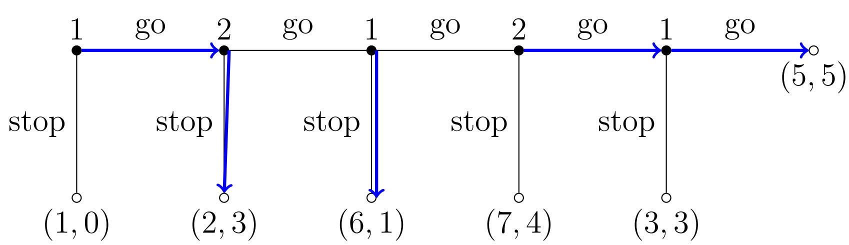
c. (1 pt.) Does either player have a strictly <u>dominant</u> strategy? If so, which player and which strategy?

No strategy is strictly dominant.

d. (1 pt.) Does either player have a strictly <u>dominated</u> strategy? If so, which player and which strategy?

Yes: Amelia's strategy Down is strictly dominated.

2. **Frenemies (4 pt. total)**



a. (1 pt.) This game has __5__ decision nodes.

b. (2 pt.) Solve this game by backward induction, shading in the action chosen at every point. (You do not have to describe the strategies in words.)

c. (1 pt.) In the subgame-perfect Nash equilibrium, the equilibrium payoffs are __(2, 3)__.

3. **True or false (6 pt. total)**

Indicate whether each of the following statements is true or false. (You don't have to explain why.)

a. (1 pt.) Every static game has at least one pure strategy Nash equilibrium. __False__

b. (1 pt.) A weakly dominant strategy is a best response against every opposing strategy. __True__

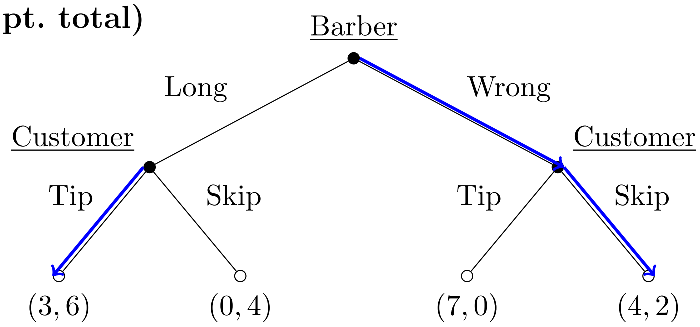
c. (1 pt.) Every dominant strategy solution is also a pure strategy Nash equilibrium. __True__

d. (1 pt.) If player 1 has a strictly dominant strategy, then the game will always have exactly one pure strategy Nash equilibrium. __False__

e. (1 pt.) The advantage of moving first is that the first mover has more information. __False__

f. (1 pt.) In game theory, each player's objective is to maximize the sum of the two players' equilibrium payoffs. __False__

4. **Barbershop blues (6 pt. total)**



a. (2 pt.) This game has __3__ subgames. The Customer has a total of __4__ different strategies ("if-then" plans) to choose from.

b. (2 pt.) Use backward induction to find the subgame-perfect Nash equilibrium. (Remember to describe each player's complete "if-then" plan.) What are the equilibrium payoffs?
See shaded lines in the game tree above. The SPNE is as follows:

Barber's strategy: "Wrong". Customer's strategy: "If Long, then Tip. If Wrong, then Skip".

In equilibrium, Barber plays Wrong and Customer plays Skip. Equilibrium payoffs are $(4, 2)$.

c. (2 pt.) Consider the following pair of strategies:

Barber's strategy: "Wrong". Customer's strategy: "Skip no matter what".

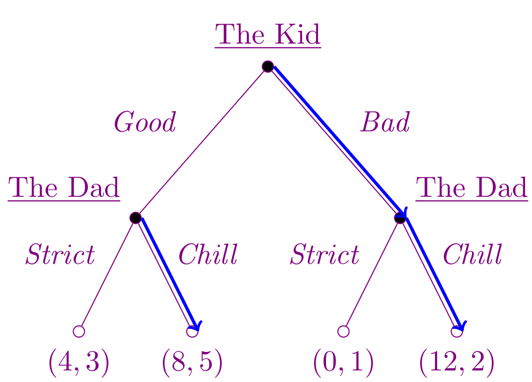
Is this a Nash equilibrium? If no, who has a profitable deviation? If yes, is it subgame-perfect?
Yes, this is a Nash equilibrium: each strategy is a best response against the other. But it's not subgame-perfect because Customer is lying about what he will do if Barber chooses Long.

5. **Up to no good? (6 pt. total)**

Early in his criminal career, the Sundance Kid is deciding whether to be a Good Kid or a Bad Kid.

The Dad

The Kid		Strict	Chill
	Good	(4), 3	8, (5)
	Bad	0, 1	(12), (2)



a. (3 pt.) Suppose this is a static game, and suppose The Kid thinks there is a 75% chance that The Dad will play "Strict" (otherwise he'll play "Chill"). Determine The Kid's best response against this mixed strategy. (Show your work.) Would Dad ever use this strategy?
If The Kid plays Good, his expected payoff is $\mathbb{E}(\text{payoff}) = \frac{3}{4} \cdot 4 + \frac{1}{4} \cdot 8 = 5$. If The Kid plays Bad, his expected payoff is $\mathbb{E}(\text{payoff}) = \frac{3}{4} \cdot 0 + \frac{1}{4} \cdot 12 = 3$. Since $5 > 3$, The Kid's best response is Good. Since Chill is strictly dominant, Dad never uses this strategy.

b. (3 pt.) Suppose The Kid moves first. In the space to the left, draw a clearly labeled game tree representing this game. If both players are rational, which action will The Kid pick? What are the equilibrium payoffs?
In this game's subgame-perfect Nash equilibrium, The Kid plays "Bad", The Dad plays "Chill no matter what", and equilibrium payoffs are (12, 2).

6. **The more the merrier? (10 pt. total)**

Demand for burritos is given by $p(Q) = 21 - 3Q$, where Q is the total amount supplied by all firms. The number of firms will change throughout this problem, but each firm can produce q burritos at a total cost $C(q) = 3q$. (Assume that all burritos are identical.)

a. (2 pt.) Suppose that the market is perfectly competitive (i.e., there are many small firms). Compute the total quantity produced (Q_{comp}) and the competitive price (p_{comp}).

We find the competitive outcome by setting the (inverse) demand equal to the marginal cost: $p(Q) = 21 - 3Q = 3 \implies Q_{comp} = 6 \implies p_{comp} = 3$.

b. (2 pt.) Now suppose that there is a single firm, which acts as a uniform-pricing monopolist. Compute the monopoly quantity (Q_{mono}) and the monopoly price (p_{mono}).

The monopolist's profit-maximization problem is:

$$\max_{Q} \ \pi = (21 - 3Q)Q - 3Q$$

The FOC is $21 - 6Q - 3 = 0 \implies Q_{mono} = 3 \implies p_{mono} = 12$.

c. (4 pt.) Now suppose there are two firms, which compete in a Cournot duopoly. Find firm 1's best-response function, $q_1^* = BR_1(\hat{q}_2)$. Then find the Nash equilibrium quantities q_1^* and q_2^*. Finally, compute the total quantity ($Q_{duo} = q_1^* + q_2^*$) and the equilibrium price (p_{duo}).

Firm 1's profit maximization problem is

$$\max_{q_1} \ \pi_1 = (21 - 3q_1 - 3\hat{q}_2)q_1 - 3q_1$$

The FOC is $21 - 6q_1 - 3\hat{q}_2 - 3 = 0 \implies q_1^* = BR_1(\hat{q}_2) = 3 - \frac{1}{2}\hat{q}_2$. Since the firms in this problem are identical/symmetric and move at the same time, firm 2's best-response function is a mirror image of firm 1's: $q_2^* = BR_2(\hat{q}_1) = 3 - \frac{1}{2}\hat{q}_1$. Setting $\hat{q}_1 = q_1^*$ and $\hat{q}_2 = q_2^*$ and solving the system of equations, we find that $q_1^* = q_2^* = 2$, so that $Q^* = 4$ and $p^* = 9$.

d. (2 pt.) Let's rank each component of social welfare under these three different market structures. Complete the table below by writing the word "largest", "smallest", or "medium" in each cell. (Use each word once per row, as I've done in the first row. You don't need to show work here.)

	uniform-pricing monopoly	Cournot duopoly	perfect competition
number of firms	smallest	medium	largest
consumer surplus	smallest	medium	largest
producer surplus	largest	medium	smallest
deadweight loss	largest	medium	smallest

7. **Follow my lead (10 pt. total)**

Two firms play a Stackelberg game in which firm 1 (the leader) chooses its quantity q_1, and then firm 2 (the follower) chooses its quantity q_2. They sell identical goods, so they receive the same price for each unit sold. Market demand is given by

$$p(Q) = 12 - Q, \qquad \text{where } Q = q_1 + q_2$$

The firms have identical cost functions, given by $C_1(q_1) = 4q_1$ and $C_2(q_2) = 4q_2$.

a. (1 pt.) In Stackelberg games, is there first-mover advantage, second-mover advantage, or neither?

Stackelberg games exhibit first-mover advantage.

b. (3 pt.) Write firm 2's profit-maximization problem in terms of q_1 and q_2. Then find firm 2's best-response function $q_2^* = BR_2(q_1)$, indicating its choice q_2^* as a function of firm 1's output.

Firm 2's profit-maximization problem is

$$\max_{q_2} \ \pi_2 = (12 - q_1 - q_2)q_2 - 4q_2$$

The FOC is $12 - q_1 - 2q_2 - 4 = 0 \implies q_2^* = BR_2(q_1) = 4 - \frac{1}{2}q_1$.

c. (4 pt.) Now find q_1^*. What is the equilibrium price, p^*? What is firm 1's profit?

Plugging firm 2's best response into firm 1's profit function, we have

$$\max_{q_1} \ \pi_1 = \left(12 - q_1 - \left(4 - \frac{1}{2}q_1\right)\right) q_1 - 4q_1 \implies \max_{q_1} \ \pi_1 = 4q_1 - \frac{1}{2}q_1^2$$

The FOC is $4 - q_1 = 0 \implies q_1^* = 4$. Firm 2 responds by choosing $q_2^* = BR_2(4) = 2$, so the total quantity is $Q^* = 6$ and the equilibrium price is $p^* = 6$. Firm 1's profit is $\pi_1 = 6 \cdot 4 - 4 \cdot 4 = 8$.

d. (2 pt.) *Note: This problem is especially challenging. Finish the rest of the exam before attempting.*

Now suppose firm 2 must pay a fixed cost $FC = 1$ if it wants to enter the market. (Firm 1 still has no fixed cost.) Assume firm 2 enters if it can make strictly positive profit (i.e., if $\pi_2 > 0$), but otherwise it stays out (if $\pi_2 \leq 0$). Find firm 1's profit-maximizing choice of output q_1^*.

We need to determine how much firm 1 must produce to ensure that firm 2 makes exactly zero profit from entering. Suppose that firm 1 produces q_1 and that firm 2 enters. In that case, firm 2 will choose $q_2^* = 4 - \frac{1}{2}q_1$, so the total quantity will be $Q^* = q_1 + 4 - \frac{1}{2}q_1 = 4 + \frac{1}{2}q_1$. The price will therefore be $p(Q^*) = 12 - (4 + \frac{1}{2}q_1) = 8 - \frac{1}{2}q_1$, and thus firm 2's profits will be

$$\pi_2 = p^* q_2^* - 4q_2^* - FC = \left(8 - \frac{1}{2}q_1\right)\left(4 - \frac{1}{2}q_1\right) - 4 \cdot \left(4 - \frac{1}{2}q_1\right) - 1 = \left(4 - \frac{1}{2}q_1\right)^2 - 1$$

This expression equals zero when $(4 - \frac{1}{2}q_1)^2 - 1 = 0 \implies q_1^* = 6$. (The other solution to this equation, $q_1^* = 10$, is invalid as it implies a negative value for q_2^*.) So, as long as firm 1 produces at least 6 units of output, firm 2 will stay out of the market, and the price will be $p^* = 6$. Choosing $q_1^* = 6$ gives firm 1 profits equal to $\pi_1 = p^* q_1^* - 4q_1^* = 6 \cdot 6 - 4 \cdot 6 = 12$.

A final detail: one can show that any other choice of q_1 yields lower profits. In particular, choosing $q_1 = 4$ (with firm 2 entering in response) only yields profits $\pi_1 = 8$ (as in part c).

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