Intermediate Microeconomic Theory
ECN 100B (Section B), Fall 2019

Professor Brendan Price

Midterm Exam #2

Name: ____________________________

ID number: __________________

- Write your answers on the exam itself, using only the space provided for each question.
  - If you run out of space for a given question, write "see extra space" in the space provided for that question, then finish your answer on the extra graded pages. Make sure to write the problem number. You may lose credit if we can't tell which question you're answering.
  - We've also included one ungraded scrap page for pure scrap work. Answers written on this ungraded page will not be graded under any circumstances.
- You must show your work on every question that requires a calculation. We will award partial credit as appropriate. Correct results without adequate work will receive little or no credit.
- Simplify all mathematical expressions as much as possible.
- The exam is graded out of 50 points. Each question is worth the indicated number of points.
- You will have 80 minutes. You must drop your pen/pencil immediately when time is up.
- As a reminder: UC Davis has a strict code of Academic Conduct. Any violations, including copying or attempting to copy from another student, will result in a score of 0.
- Good luck!

Do not turn this page until I tell you to start.

1. **It takes two to tango (8 pt. total)**

Tango's not really static, but let's pretend it is. Answer each question based on the payoff matrix.

		Morgan	
		Giro	Ocho
Ada	Cabeceo	⑦, 3	1, ⑤
	Corrida	5, 2	−1, ③
	Cruzada	−5, ③	④, ③

a. (4 pt.) Circle every payoff that corresponds to a player's best response. Then identify all pure strategy Nash equilibria. (If there aren't any, write "none".)

One PSNE: (Cruzada, Ocho).

b. (2 pt.) Ada's best response against Giro is Cabaceo. Morgan's best response against Corrida is Ocho.

c. (1 pt.) Does either player have a weakly <u>dominant</u> strategy? If so, which player and which strategy?

Yes: Morgan's strategy Ocho is weakly dominant.

d. (1 pt.) Does either player have a strictly <u>dominated</u> strategy? If so, which player and which strategy?

Yes: Ada's strategy Corrida is strictly dominated.

2. **True or false (6 pt. total)**

Indicate whether each of the following statements is true or false. (You don't have to explain why.)

a. (1 pt.) Every pure strategy is also considered a mixed strategy. True

b. (1 pt.) In game theory, each player's objective is to beat her opponent by getting a higher payoff than the other player gets. False

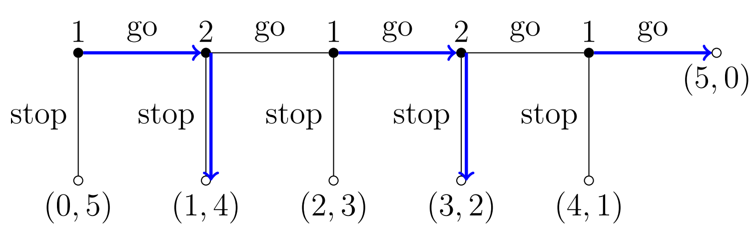
c. (1 pt.) If a static game has a dominant strategy solution, then it must have exactly one pure strategy Nash equilibrium. True

d. (1 pt.) The advantage of moving second is that the second mover can commit to playing a particular action. False

e. (1 pt.) A weakly dominant strategy is never a best response. False

f. (1 pt.) If a player flips a coin to decide whether she'll choose action A or action B, then she must be indifferent between A and B. True

3. **Well, it *looks* like a centipede . . . (4 pt. total)**

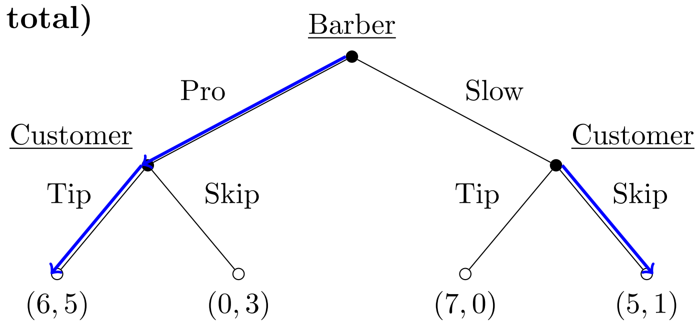


a. (1 pt.) This game has 5 subgames.

b. (2 pt.) Solve this game by backward induction, shading in the action chosen at every point. (You do not have to describe the strategies in words.)

c. (1 pt.) In the subgame-perfect Nash equilibrium, the equilibrium payoffs are (1, 4).

4. **Amateur hour (6 pt. total)**



a. (2 pt.) This game has __3__ decision nodes. The Customer has a total of __4__ different strategies ("if-then" plans) to choose from.

b. (2 pt.) Use backward induction to find the subgame-perfect Nash equilibrium. (Remember to describe each player's complete "if-then" plan.) What are the equilibrium payoffs?

See shaded lines in the game tree above. The SPNE is as follows:

Barber's strategy: "Pro". Customer's strategy: "If Pro, then Tip. If Slow, then Skip".

In equilibrium, Barber plays Pro and Customer plays Tip. Equilibrium payoffs are $(6, 5)$.

c. (2 pt.) Consider the following pair of strategies:

Barber's strategy: "Pro". Customer's strategy: "Tip no matter what".

Is this a Nash equilibrium? If yes: is it subgame-perfect? If no: who isn't playing a best response?

No, not a Nash equilibrium: given Customer's strategy, Barber's best response is to play Slow.

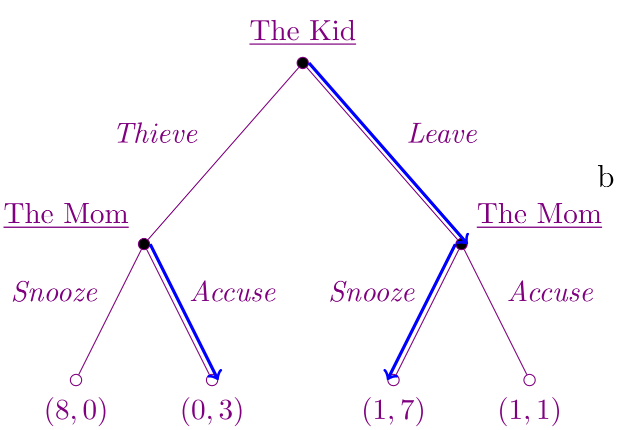
5. **Petty theft (6 pt. total)**

Early in his criminal career, The Sundance Kid is deciding whether to raid the family cookie jar.

		The Mom	
		Snooze	Accuse
The Kid	Thieve	(8), 0	0, (3)
	Leave	1, (7)	(1), 1

a. (3 pt.) Suppose this is a static game, and suppose The Kid thinks there's a 25% chance that Mom will play "Snooze" (otherwise she'll play "Accuse"). Determine The Kid's best response to this mixed strategy. (Show your work.) What is The Kid's expected payoff?

If The Kid plays Thieve, his expected payoff is $\mathbb{E}(\text{payoff}) = \frac{1}{4} \cdot 8 + \frac{3}{4} \cdot 0 = 2$. If he plays Leave, he gets 1 no matter what, so his expected payoff is 1. Since $2 > 1$, The Kid's best response is Thieve. His expected payoff is 2.



b. (3 pt.) Suppose The Kid moves first. In the space to the left, draw a clearly labeled game tree representing this game. If both players are rational, which action will The Kid pick? What are the equilibrium payoffs?

In this game's SPNE, The Kid plays "Leave". (The Mom plays "If he Thieves, I'll Accuse; if he Leaves, I'll Snooze".) Equilibrium payoffs: (1, 7).

6. **Price-takers and price-makers (10 pt. total)**

Demand for taxi rides is given by $p(Q) = 48 - 3Q$, where Q is the total amount supplied by all firms. The number of firms will change throughout this problem, but each firm can supply q rides at a total cost $C(q) = 12q$. (All companies offer rides of identical comfort and quality.)

a. (2 pt.) Suppose that the market is perfectly competitive (i.e., there are many small firms). Compute the total quantity produced (Q_{comp}) and the competitive price (p_{comp}).

We find the competitive outcome by setting the (inverse) demand equal to the marginal cost: $p(Q) = 48 - 3Q = 12 \implies Q_{comp} = 12 \implies p_{comp} = 12$.

b. (2 pt.) Now suppose there is only one firm (Uber), which acts as a uniform-pricing monopolist. Compute the monopoly quantity (Q_{mono}) and the monopoly price (p_{mono}).

The monopolist's profit-maximization problem is:

$$\max_{Q} \ \pi = (48 - 3Q)Q - 12Q$$

The FOC is $48 - 6Q - 12 = 0 \implies Q_{mono} = 6 \implies p_{mono} = 30$.

c. (4 pt.) Now suppose there are two firms (Uber and Lyft), which compete in a Cournot duopoly. Find firm 1's best-response function, $q_1^* = BR_1(\hat{q}_2)$. Then find the Nash equilibrium quantities q_1^* and q_2^*. Finally, compute the total quantity ($Q_{duo} = q_1^* + q_2^*$) and the equilibrium price (p_{duo}).

Firm 1's profit maximization problem is

$$\max_{q_1} \ \pi_1 = (48 - 3q_1 - 3\hat{q}_2)q_1 - 12q_1$$

The FOC is $48 - 6q_1 - 3\hat{q}_2 - 12 = 0 \implies q_1^* = BR_1(\hat{q}_2) = 6 - \frac{1}{2}\hat{q}_2$. Since the firms in this problem are identical/symmetric and move at the same time, firm 2's best-response function is a mirror image of firm 1's: $q_2^* = BR_2(\hat{q}_1) = 6 - \frac{1}{2}\hat{q}_1$. Setting $\hat{q}_1 = q_1^*$ and $\hat{q}_2 = q_2^*$ and solving the system of equations, we find that $q_1^* = q_2^* = 4$, so that $Q^* = 8$ and $p^* = 24$.

d. (2 pt.) Let's explore how social welfare varies across these three different market structures. Complete the table below by writing the word "largest", "smallest", or "medium" in each cell. (Use each word once per row, as I've done in the first row. You don't need to show work here.)

	perfect competition	uniform-pricing monopoly	Cournot duopoly
number of firms	largest	smallest	medium
consumer surplus	largest	smallest	medium
price markup	smallest	largest	medium
deadweight loss	smallest	largest	medium

7. **Who's on first, What's on second (10 pt. total)**

Two firms play a Stackelberg game in which firm 1 (the leader) chooses its quantity q_1, and then firm 2 (the follower) chooses its quantity q_2. They sell identical goods, so they receive the same price for each unit sold. Market demand is given by

$$p(Q) = 20 - Q, \qquad \text{where } Q = q_1 + q_2$$

The firms have identical cost functions, given by $C_1(q_1) = 4q_1$ and $C_2(q_2) = 4q_2$.

a. (1 pt.) In Stackelberg games, is there first-mover advantage, second-mover advantage, or neither?
Stackelberg games exhibit first-mover advantage.

b. (3 pt.) Write firm 2's profit-maximization problem in terms of q_1 and q_2. Then find firm 2's best-response function $q_2^* = BR_2(q_1)$, indicating its choice q_2^* as a function of firm 1's output.
Firm 2's profit-maximization problem is

$$\max_{q_2} \ \pi_2 = (20 - q_1 - q_2)q_2 - 4q_2$$

The FOC is $20 - q_1 - 2q_2 - 4 = 0 \implies q_2^* = BR_2(q_1) = 8 - \frac{1}{2}q_1$.

c. (4 pt.) Now find q_1^*. What is the equilibrium price, p^*? What is firm 1's profit?
Plugging firm 2's best response into firm 1's profit function, we have

$$\max_{q_1} \ \pi_1 = \left(20 - q_1 - \left(8 - \frac{1}{2}q_1\right)\right) q_1 - 4q_1 \implies \max_{q_1} \ \pi_1 = 8q_1 - \frac{1}{2}q_1^2$$

The FOC is $8 - q_1 = 0 \implies q_1^* = 8$. Firm 2 responds by choosing $q_2^* = BR_2(8) = 4$, so total output is $Q^* = 12$ and the equilibrium price is $p^* = 8$. Firm 1's profit is $\pi_1 = 8 \cdot 8 - 4 \cdot 8 = 32$.

d. (2 pt.) *Note: This problem is especially challenging. Finish the rest of the exam before attempting.*

Now suppose firm 2 must pay a fixed cost $FC = 9$ if it wants to enter the market. (Firm 1 still has no fixed cost.) Assume firm 2 enters if it can make strictly positive profit (i.e., if $\pi_2 > 0$), but otherwise it stays out (if $\pi_2 \leq 0$). Find firm 1's profit-maximizing choice of output q_1^*.

We need to determine how much firm 1 must produce to ensure that firm 2 makes exactly zero profit from entering. Suppose that firm 1 produces q_1 and that firm 2 enters. In that case, firm 2 will choose $q_2^* = 8 - \frac{1}{2}q_1$, so the total quantity will be $Q^* = q_1 + 8 - \frac{1}{2}q_1 = 8 + \frac{1}{2}q_1$. The price will therefore be $p(Q^*) = 20 - (8 + \frac{1}{2}q_1) = 12 - \frac{1}{2}q_1$, and thus firm 2's profits will be

$$\pi_2 = p^* q_2^* - 4q_2^* - FC = (p^* - 4)q_2^* - FC = \left(8 - \frac{1}{2}q_1\right)\left(8 - \frac{1}{2}q_1\right) - 9 = \left(8 - \frac{1}{2}q_1\right)^2 - 9$$

This expression equals zero when $(8 - \frac{1}{2}q_1)^2 - 9 = 0 \implies q_1^* = 10$. (The other solution with $q_1^* = 22$ is invalid as it yields a negative price.) So, as long as firm 1 produces at least 10 units of output, firm 2 will stay out of the market, and the price will be $p^* = 10$. Choosing $q_1^* = 10$ gives firm 1 profits equal to $\pi_1 = p^* q_1^* - 4q_1^* = 10 \cdot 10 - 4 \cdot 10 = 60$.
A final detail: one can show that any other choice of q_1 yields lower profits. In particular, choosing $q_1 = 8$ (with firm 2 entering in response) only yields profits $\pi_1 = 32$ (as in part c).

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