Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #1

Due: Friday, October 4th at 5:00pm

This homework assignment will be graded 100% on the basis of effort. In future homework assignments, some questions (accounting for 50% of the total points) will be fully graded, and others (the remaining 50% of points) will be graded on the basis of effort.

1 Bubble tax (18 pts.)

Throughout this problem (and throughout the first part of the course), "total surplus" is defined as "consumer surplus + producer surplus + government tax revenue (if any)".

Davis's bubble-tea market is fiercely competitive. Suppose that demand for bubble tea is given by $Q_D(p) = 300 - 30p$, and supply is given by $Q_S(p) = 60p - 60$.

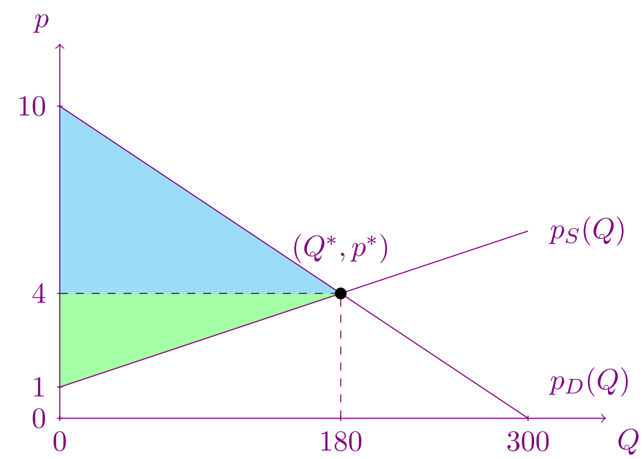
a. (3 pts.) Find the equilibrium price p^* and quantity Q^*.

For the market to clear, the quantity supplied must equal the quantity demanded:

$$Q_D(p) = Q_S(p) \implies 300 - 30p = 60p - 60 \implies p^* = 4 \implies Q^* = 180$$

b. (3 pts.) Draw a carefully labeled graph representing this market. (Since price goes on the vertical axis, you'll need to invert the demand and supply curves.) Label the Q and p axes wherever the supply and demand curves intersect each other or the axes. Also label the equilibrium outcome (Q^*, p^*).

Solving each equation for price, we obtain the inverse demand curve $p_D(Q) = 10 - \frac{1}{30}Q$ and the inverse supply curve $p_S(Q) = 1 + \frac{1}{60}Q$. Here's the graph:



c. (3 pts.) Calculate the consumer surplus, producer surplus, and deadweight loss (if any). What is the total surplus in this market?

Consumer surplus (shown in blue) is $\frac{1}{2}(10-4)(180) = 540$. Producer surplus (shown in green) is $\frac{1}{2}(4-1)(180) = 270$. There is no deadweight loss: all desirable transactions—those where the consumer's willingness to pay exceeds the producer's marginal cost of production—take place in equilibrium. Total surplus is $540 + 270 = 810$.

d. (3 pts.) What is the most any consumer is willing to pay for bubble tea in this market? At the equilibrium price, how much is the *marginal* consumer willing to pay? How much social surplus is created by the marginal transaction? (Remember: the marginal consumer is the one who is indifferent between buying and not buying.)

The most any consumer is willing to pay is 10, since the quantity demanded falls to zero if the price exceeds this level. The marginal consumer is willing to pay at most 4, which is the market price. The marginal transaction generates zero social surplus: the buyer's willingness to pay exactly equals the producer's marginal cost of production.

Now suppose that, to finance an exciting new Bicycle-Themed Sculpture Fund, the city of Davis imposes a \$3 tax on bubble-tea shops for each unit of tea they sell.

e. (3 pts.) Find the new equilibrium price and quantity.

Since the tax is levied on producers, the supply curve shifts upwards by the amount of the tax. Why? Well, each point on the supply curve tells us the "reservation price" of the relevant producer, the lowest price that she would be willing to accept from a buyer. If I run a convenience store, and I know I will have to pay a \$3 tax once the transaction is completed, my reservation price will rise by \$3 to offset my tax liability.

The new (inverse) supply curve is therefore

$$p_S(Q) = 1 + \frac{1}{60}Q + 3 = 4 + \frac{1}{60}Q$$

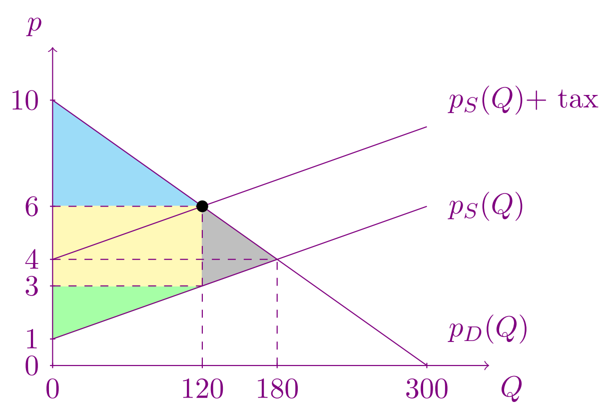
What about the demand curve? The demand curve tells us how much the relevant consumer is willing to pay to buy the good. Since buyers don't *directly* pay the cost of the tax, the demand curve is unchanged: buyers just care about the price they're charged, not about whether the seller has to pay taxes afterwards.

In equilibrium, the (original) demand curve must intersect the (shifted) supply curve:

$$10 - \frac{1}{30}Q^* = 4 + \frac{1}{60}Q^* \implies Q^* = 120 \implies p^* = 6$$

f. (3 pts.) Calculate consumer surplus, producer surplus, tax revenue, and deadweight loss under this tax. How has total surplus changed compared to part c? How does the change in total surplus relate to the change in deadweight loss?

I didn't ask you to draw another graph here, but the graph is helpful:



Computing the four terms:

- Consumer surplus (blue) $= \frac{1}{2}(10-6)120 = 240$
- Producer surplus (green) $= \frac{1}{2}(3-1)120 = 120$
- Tax revenue (yellow) $= (6-3)120 = 360$
- Deadweight loss (gray) $= \frac{1}{2}(6-3)(180-120) = 90$

Notice that both consumer and surplus have fallen: although buyers don't *directly* pay for this tax, they pay for it *indirectly* because the equilibrium price has gone up. The "economic incidence" of the tax (who really pays) is shared by both sides.

Total surplus has fallen from 810 to 720 (= 240 CS + 120 PS + 360 TR). The reduction in total surplus is 90, exactly equal to the deadweight loss. This is no coincidence: we *define* deadweight loss as the loss in surplus relative to the socially optimal allocation.

2 Practice makes perfect (12 pts.)

Solve each of the following optimization problems. (You should watch Lecture 2 first.)

a. (4 pts.) A drug company sells an over-the-counter medication in a perfectly competitive market, where the price is $p = 24$. For each of the following cost functions, write down the company's profit-maximization problem, and then find q^*.

i. $C(q) = 2q^3$

The profit-maximization problem is

$$\max_q \ \pi(q) = 24q - 2q^3 \implies \text{FOC: } 24 - 6(q^*)^2 = 0 \implies q^* = 2$$

Since $\frac{d^2\pi(q)}{dq^2} = -12q < 0$ for all $q > 0$, the profit function is concave, which confirms that $q^* = 2$ is the unique global maximum.

ii. $C(q) = 30q + q^2$

The profit-maximization problem is

$$\max_q \ \pi(q) = 24q - 30q - q^2$$

The FOC gives us $24 - 30 - 2q = 0 \implies q^* = -3$. But the firm can't choose a negative quantity, so this is an invalid solution. In fact, $\frac{d\pi(q)}{dq} < 0$ for any non-negative choice of output q, so the more medication the firm produces, the more money it loses. The firm's profit-maximizing choice is therefore $q^* = 0$.

iii. $C(q) = 10q$

The profit-maximization problem is

$$\max_q \ \pi(q) = 24q - 10q = 14q$$

Here, we can see right away that profits are an increasing function of quantity. (If we try to take the FOC, we get "$14 = 0$", indicating that there is no interior solution.) The more the firm produces, the more money it makes, so the firm would like to set $q^* = \infty$. (Technically, the solution is undefined, but it's reasonable to think of the firm as wanting to produce an infinite level of output in this case.)

iv. $C(q) = 24q$

The profit-maximization problem is

$$\max_q \ \pi(q) = 24q - 24q = 0$$

so no matter what it does, the firm will make zero profit. Therefore, there is no unique optimal choice: *any* choice of q^* is an optimum.

b. (4 pts.) A firm chooses q to solve the profit-maximization problem below:

$$\max_{q} \ \pi(q) = (10 - q)q - 2q$$

i. Use the first-order condition to find a "candidate" solution q^*.

We can start by combining terms to simplify the problem:

$$\max_{q} \ \pi(q) = 8q - q^2$$

The FOC gives $8 - 2q = 0 \implies q^* = 4$.

ii. Use the second-order condition to verify that q^* is the optimal choice.

The second derivative of the profit function with respect to q is

$$\frac{d^2\pi(q)}{dq^2} = -2 < 0$$

Since the profit function is globally concave, our candidate solution $q^* = 4$ is indeed the optimal choice.

iii. Find $\pi(q^*)$.

Plugging $q^* = 4$ into the profit function gives

$$\pi(q^*) = (10 - 4)(4) - 2(4) = 16$$

c. (4 pts.) A restaurant sells a mixture of sit-down meals (q_1) and take-out meals (q_2). The market for each meal type is perfectly competitive, with prices $p_1 = 18$ and $p_2 = 8$, respectively, and the restaurant's cost function is $C(q_1, q_2) = 3q_1^2 + q_2^2$.

The restaurant's profit-maximization problem is therefore

$$\max_{q_1, q_2} \ \pi(q_1, q_2) = 18q_1 + 8q_2 - 3q_1^2 - q_2^2$$

i. Find q_1^*, q_2^*, and profits $\pi(q_1^*, q_2^*)$.

Since there are two choice variables, we will need two FOCs. The FOC for q_1 is

$$\frac{\partial}{\partial q_1}\pi(q_1, q_2) = 18 - 6q_1^* = 0 \implies q_1^* = 3$$

And the FOC for q_2 is

$$\frac{\partial}{\partial q_2}\pi(q_1, q_2) = 8 - 2q_2^* = 0 \implies q_2^* = 4$$

Total profits are

$$\pi(q_1^*, q_2^*) = (18)(3) + (8)(4) - (3)(3^2) - (4^2) = 54 + 32 - 27 - 16 = 43$$

ii. Now suppose that a loud construction project reduces demand for sit-down meals in the area, so that p_1 goes down. Assuming that p_2 remains unchanged, what happens to the restaurant's optimal quantity of take-out meals (q_2^*)?

Nothing! Let's write the profit-maximization problem in terms of an unspecified price p_1 (rather than the particular price $p_1 = 18$):

$$\max_{q_1, q_2} \pi(q_1, q_2) = p_1 q_1 + 8q_2 - 3q_1^2 - q_2^2$$

The FOC for q_2 is still $8 - 2q_2 = 0 \implies q_2^* = 4$. So the optimal choice q_2^* is independent of the price of sit-down meals. (In case you are wondering, this is not *usually* true in problems with two choice variables. We will see examples to the contrary in future problems.)