Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #2

Due: Friday, October 11th at 5:00pm

# 1 Monopoly, naturally (12 pts.)

Because producing and distributing electricity involves big fixed costs and therefore big "economies of scale", it doesn't make economic sense to have multiple electrical utilities serving the same geographic area. Instead, state governments let electric utilities operate as "natural monopolies", while placing restrictions on what prices they are allowed to charge.

An electrical utility faces demand given by $p(Q) = 36 - Q$. Its cost function is $C(Q) = Q^2$. (To keep things simple, we'll ignore the fixed costs throughout this problem.) Assume that the utility is legally obligated to charge all consumers the same price.

a. (3 pts.) Suppose that the firm chooses quantity. Express the firm's profits as a function of $Q$. Solve for the monopoly quantity $Q_m$, then solve for the monopoly price $p_m$.

In general, a uniform-pricing monopolist's profit function is $\pi(Q) = p(Q)Q - C(Q)$. So the profit-maximization problem is

$$\max_Q \ \pi(Q) = (36 - Q)Q - Q^2 \qquad \text{or, simplifying,} \qquad \max_Q \ \pi(Q) = 36Q - 2Q^2$$

Differentiating with respect to $Q$, we obtain the FOC $36 - 4Q_m = 0$, which implies $Q_m = 9$ and therefore $p_m = 27$.

b. (3 pts.) Now suppose that the firm chooses price. Express profits as a function of $p$. Solve for the monopoly price $p_m$, then solve for the monopoly quantity $Q_m$. How do your answers compare with the ones you found in part a?

By inverting the demand curve, we can express the quantity demanded as a function of the price: $Q(p) = 36 - p$. In general, we can write the firm's profits as $\pi(p) = pQ(p) - C(Q(p))$, so the profit-maximization problem is

$$\max_p \ \pi(p) = p(36 - p) - (36 - p)^2 \qquad \text{or} \qquad \max_p \ \pi(p) = 108p - 2p^2 - 1296$$

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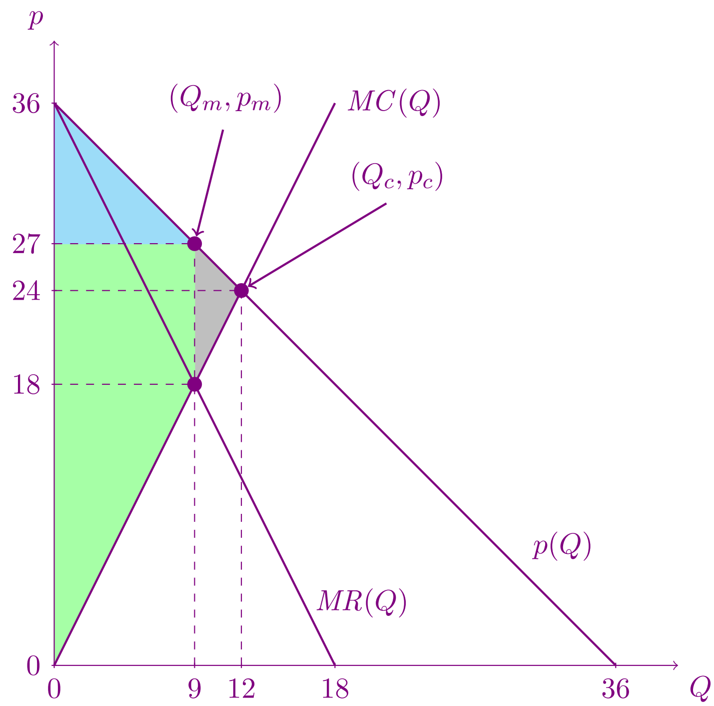
Differentiating with respect to $p$, we obtain the FOC $108 - 4p_m = 0$, which implies $p_m = 27$ and therefore $Q_m = 9$. As we'd expect, we get the same exact solutions in parts a and b.

c. (3 pts.) Draw a clearly labeled graph representing this market. Be sure to draw the demand curve, marginal cost curve, and marginal revenue curve. Label the monopoly solution ($Q_m$, $p_m$). Calculate the producer surplus, consumer surplus, and DWL.




The producer surplus (green) equals 162 (the triangle part has area 81, and the square part has area 81 too). The consumer surplus (blue) is 40.5. The DWL (gray) is 13.5.

d. (3 pts.) Explain how the government can use a price regulation to achieve the socially optimal level of output in this market. Should we use a price ceiling or a price floor? At what price should we set the ceiling/floor? Compute consumer surplus, producer surplus, and DWL under this regulatory policy.

The socially optimal level of output is the amount produced under perfect competition. In the graph above, we can see that the demand curve intersects the marginal cost curve at the point $Q_c = 12$, $p_c = 24$. The government can achieve this outcome by setting a price ceiling of $\underline{p} = 24$. Under this policy, consumer surplus rises to 72, producer surplus falls to 144, and the DWL falls to 0. Relative to the monopoly outcome, total surplus has increased from 202.5 to 216. Notice that—as we've seen in previous problems—the increase in total surplus is exactly equal to the decline in DWL.

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## 2 Pricey pills (6 pts.)

When the patent on a popular medication expires, other companies quickly introduce generic versions that are therapeutically equivalent to the original version. Although doctors regard generic medications as being just as effective, many consumers show "brand loyalty": they are willing to pay more for brand-name drugs like Tylenol than for generic acetaminophen. This gives drug companies a bit of market power even after their patents expire.

a. (3 pts.) Demand for Tylenol is given by $p(Q) = 60 - 6Q$. Compute the elasticity of demand as a function of $Q$, simplifying your answer as much as possible. For what value of $Q$ is demand perfectly inelastic? For what value of $Q$ is demand unit elastic? For what value of $Q$ is demand perfectly elastic?

Since $Q(p) = 10 - \frac{1}{6}p$, the elasticity of demand is

$$\varepsilon = \frac{dQ}{dp}\frac{p}{Q} = \left(-\frac{1}{6}\right)\left(\frac{60-6Q}{Q}\right) = 1 - \frac{10}{Q}.$$

Demand is perfectly inelastic when $\varepsilon = 0$, which occurs when $Q = 10$. Demand is unit elastic when $\varepsilon = -1$, which occurs when $Q = 5$. Demand is perfectly elastic when $\varepsilon = -\infty$, which occurs when $Q = 0$.

b. (3 pts.) Demand for Advil is given by $Q(p) = 100p^{-5}$. Compute the elasticity of demand. (Hint: this is a special function for which the elasticity doesn't vary with $Q$.) If Advil's manufacturer is profit-maximizing, what price markup will it choose?

The exponents are a little tricky here, but the elasticity is

$$\frac{dQ}{dp}\frac{p}{Q} = \left(100 \cdot -5 \cdot p^{-6}\right)\left(\frac{p}{100p^{-5}}\right) = -5 \cdot \frac{100}{100} \cdot \frac{p^{-5}}{p^{-5}} = -5$$

Plugging the elasticity into the markup equation gives

$$\frac{p^* - MC}{MC} = -\frac{1}{1+\varepsilon} = -\frac{1}{1-5} = 0.25,$$

so the markup equals 25%.

c. (Optional and ungraded) A uniform-pricing monopolist firm faces a downward-sloping (and differentiable) demand curve $p(Q)$. Prove that its total revenue is maximized at a point where demand is unit elastic. Provide economic intuition for this result.

Let's find the quantity that maximizes total revenue:

$$\max_{Q} \; R(Q) = p(Q)Q$$

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The FOC sets the marginal revenue equal to zero:

$$\frac{dR(Q)}{dQ} = p(Q) + p'(Q)Q = 0 \implies p'(Q)Q = -p(Q)$$

Let's write $p'(Q)$ as $\frac{dp}{dQ}$ and divide both sides by $p$:

$$\frac{dp}{dQ}\frac{Q}{p} = -1$$

But the lefthand side of this equation is the inverse price elasticity. We can obtain the regular elasticity by taking the reciprocal of both sides:

$$\frac{dQ}{dp}\frac{p}{Q} = \frac{1}{-1} = -1 \implies \varepsilon = -1$$

which proves that demand is unit elastic at the revenue-maximizing quantity.

What's the intuition for this result? When demand is relatively elastic ($\varepsilon < -1$), a small decrease in price will produce a bigger increase in the quantity demanded, so that total revenue will increase. When demand is relatively inelastic ($-1 < \varepsilon < 0$), a small increase in price will produce an even smaller decrease in quantity demanded, so that total revenue once again goes up. So, if we start from any point where demand is *not* unit elastic, we can find a revenue-increasing deviation.

By contrast: when demand is unit elastic, a small change in the price results in an offsetting, equally sized change in quantity demanded, so that total revenue doesn't change. Revenue is maximized at this point.

## 3 Personalized medicine? (12 pts.)

Suppose that the pharmaceutical firm Merck is deciding whether to develop a new diagonistic procedure that can detect early-stage Alzheimer's disease more accurately than existing tests.

Developing this technology would require an up-front fixed cost $FC > 0$. If Merck develops the technology, it can screen $Q$ patients for Alzheimer's at the variable cost $VC(Q) = 20Q$. Merck estimates that market demand for the procedure would be $p(Q) = 80 - \frac{1}{10}Q$.

a. (3 pts.) Suppose that other companies can quickly copy Merck's procedure as soon as it is developed, so that the market for medical tests will become perfectly competitive. If Merck develops the procedure, what are the equilibrium price $p_c$ and quantity $Q_c$? If $FC = 5000$, will Merck develop the procedure? What about if $FC = 10{,}000$?

In a competitive market, we find the point where the market demand curve equals the market supply curve (which is the same as the marginal cost curve):

$$80 - \frac{1}{10}Q_c = 20 \implies Q_c = 600 \implies p_c = 20$$

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In this case, Merck's profits from developing the procedure would be

$$\pi(q_c^{\text{Merck}}) = p_c q_c^{\text{Merck}} - VC(q_c^{\text{Merck}}) - FC = 20q_c^{\text{Merck}} - 20q_c^{\text{Merck}} - FC = -FC$$

where $q_c^{\text{Merck}}$ is the amount that Merck itself would produce in a competitive market. (In a competitive market, Merck isn't a monopolist, so it only produces a portion of the overall market quantity.) Regardless of how many units $q_c^{\text{Merck}}$ produces in this case, its profits would equal $-FC$. If $FC > 0$, developing the technology would yield negative profits, so Merck will not develop the procedure if either $FC = 5000$ or $FC = 10{,}000$.

b. (3 pts.) Now suppose that, if Merck develops the procedure, it will receive a patent that allows it to operate as a uniform-pricing monopolist. In this case, if Merck develops the procedure, how many patients will it screen ($Q_m$), and what will it charge ($p_m$)? If $FC = 5000$, will Merck develop the procedure? What about if $FC = 10{,}000$?

If the procedure has been developed, Merck solves the profit-maximization problem

$$\max_Q \ \pi(Q) = \underbrace{(80 - {}^1\!/_{10}Q)\,Q}_{\text{total revenue}} - \underbrace{(20Q + FC)}_{\text{total costs}}$$

Let's simplify this expression before we take the derivative:

$$\max_Q \ \pi(Q) = 60Q - \frac{1}{10}Q^2 - FC$$

The FOC is

$$60 - \frac{1}{5}Q_m = 0 \implies Q_m = 300 \implies p_m = 80 - \frac{1}{10} \cdot 300 = 50$$

Merck's profits from developing the procedure are therefore

$$\pi(Q_m) = 50 \cdot 300 - 20 \cdot 300 - FC = 9000 - FC$$

If $FC = 5000$, then developing the procedure yields positive profit ($\pi = 4000$), so yes, Merck will develop the procedure. If $FC = 10{,}000$, developing the procedure would yield negative profit ($\pi = -1000$), so Merck will not do so.

c. (3 pts.) Now suppose that, if Merck develops the procedure, it is legally permitted (and able) to engage in perfect price discrimination. If Merck develops the procedure, what are its optimal quantity $Q_{ppd}$, revenue $R(Q_{ppd})$, and variable costs $VC(Q_{ppd})$? If $FC = 5000$, will Merck develop the procedure? What about if $FC = 10{,}000$?

If Merck engages in perfect price discrimination, it will sell the test to all consumers whose willingness to pay is at least \$20, which means that

$$80 - \frac{1}{10}Q_{ppd} = 20 \implies Q_{ppd} = 600$$

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(This is the same output that we would see in a competitive market.) Merck's revenue in this case is the area under the demand curve from $Q = 0$ to $Q = 600$, which is the sum of a triangle and a rectangle. We can graph the market and use geometric rules to compute the area, but here's how we can do the same calculation using integrals:

$$R(Q_{ppd}) = \int_0^{600} \left(80 - \frac{1}{10}Q\right) dQ = \left(80Q - \frac{1}{20}Q^2\right)\Big|_0^{600} = 48{,}000 - 18{,}000 = 30{,}000$$

Its variable costs are $VC(Q_{ppd}) = 20 \cdot 600 = 12{,}000$. Merck's profits are therefore

$$\pi = 30{,}000 - 12{,}000 - FC = 18{,}000 - FC$$

So if either $FC = 5000$ or $FC = 9000$, $\pi > 0$ and Merck will develop the procedure.

d. (3 pts.) Suppose that $FC = 5000$. Using your answers above, compute consumer surplus, producer surplus, and total surplus under each of the following policies:

i. No patent protecting Merck's innovation (as in part a).

ii. A patent letting Merck to operate as a uniform-pricing monopolist (as in b).

iii. Legal permission for Merck to engage in perfect price discrimination (as in c).

(If Merck develops the procedure, make sure to subtract $FC$ from the producer surplus.) If we are trying to maximize total surplus, which of these policies is best? If we are instead trying to maximize *consumer* surplus, which policy is best?

Under the no-patent policy, Merck knows it can't recoup its investment if it develops the technology, so it stays out of the market. Consumer, producer, and total surplus all equal zero. Under the uniform-pricing patent, Merck does develop the procedure. Consumer surplus equals 4500, producer surplus equals 4000, and total surplus equals 8500. Under the perfect-price-discrimination policy, consumer surplus equals 0, producer surplus equals 13,000, and total surplus equals 13,000. If our goal is to maximize total surplus, we should let Merck engage in perfect price discrimination, in which case Merck gets all of the available gains from trade. If our goal is to maximize consumer surplus, the uniform-pricing patent is a better policy.

One final observation: this problem illustrates the basic tradeoff between giving companies incentives to innovate, on the one hand—by giving them patent protection or other favorable legal treatment that allows them to recoup their up-front investments—and trying to ensure that consumers get some of the gains from trade. Even though letting Merck price discriminate is Pareto efficient, in practice we often care about "equity" as well, and equity considerations may lead us to conclude that the uniform-pricing patent is preferable for society.

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