Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #3

Due: Friday, October 18th at 5:00pm

1 Expensive clients (12 pts.)

A barber shop offers haircuts to both students and faculty. Student demand for haircuts is given by $p_S(Q_S) = 24 - \frac{1}{4}Q_S$. Faculty demand for haircuts is given by $p_F(Q_F) = 24 - \frac{1}{2}Q_F$. Students have more hair than professors (even the young professors), and longer hair costs more to cut. Reflecting this fact, the barber shop's total costs are

$$C(Q_S, Q_F) = 16Q_S + 10Q_F$$

Suppose first that the barber shop can engage in perfect price discrimination.

a. (3 pts.) How many students get haircuts (Q_S^*)? How many faculty get haircuts (Q_F^*)? How much profit will the barber shop make?

b. (3 pts.) Under perfect price discrimination, is each of these statements true or false? Briefly explain your reasoning.

   i. Every faculty member with positive willingness to pay ends up getting a haircut.

   ii. Among the people who get haircuts, students pay more than faculty on average.

   iii. The cheapest haircut sold is sold to a faculty member

Now suppose that the barbershop cannot engage in personalized pricing. However, it is able to offer one price for students and a different price for faculty.

c. (3 pts.) Find the monopoly's profit-maximizing prices p_S^* and p_F^* under group price discrimination. Which group is charged a bigger price markup?

Upset about discriminatory prices, student groups organize protests against the barber shop, using the catchy slogan "It's unfair / to tax our hair!" The protests go viral, and the barber shop reluctantly agrees to charge everybody the same price, regardless of cost.

d. (3 pts.) Compute the market demand curve $Q(p)$, then write the barber shop's profits as a function of p. (Be careful with the costs!) What price will the barber shop charge?

2 Expensive tastes (9 pts.)

FancyPants Vineyard sells bottles of wine both to tourists who come for wine tastings and to foreign wholesalers. Demand from tourists is given by

$$p_T(Q_T) = 48 - Q_T$$

Wholesale demand is perfectly elastic at price $p_W = 32$. Fancypants's total costs are

$$C(Q_T, Q_W) = (Q_T + Q_W)^2$$

where Q_T and Q_W are the quantities sold to tourists and wholesalers, respectively.

a. (3 pts.) Write FancyPants's profit function in terms of Q_T and Q_W. What is its total tourist revenue, expressed as a function of Q_T? What is its total wholesaler revenue, expressed as a function of Q_W?

b. (3 pts.) Solve for FancyPants's optimal quantities Q_T^* and Q_W^*.

c. (3 pts.) A trade war disrupts FancyPants's access to foreign markets, so that it has to choose $Q_W^* = 0$. Will Q_T^* increase, decrease, or stay the same? What about p_T^*? What about FancyPants's profits?

3 Expensive coffee (9 pts.)

Mishka's Cafe produces coffee using a mixture of labor and capital, with the production function $q(L, K) = 10\sqrt{LK}$. In the short run, however, its capital stock is fixed at the level $\overline{K} = 4$, so that Mishka's has a short-run production function $q(L) = 20\sqrt{L}$.

a. (3 pts.) Suppose that Mishka's is both a price-taker and a wage-taker, facing an output price $p = 3$ and a wage rate $w = 10$.

   i. Compute the marginal physical product of labor as a function of L.

   ii. Compute the marginal revenue product of labor as a function of L.

   iii. Compute the profit-maximizing choice of labor L^*.

b. (3 pts.) Mishka's is still a wage-taker (with $w = 10$), but now suppose that it faces downward sloping demand for coffee, given by $p(q) = 10 - \frac{1}{10}q$.

   i. Write the profit as a function of L. (No "q" terms should appear in your answer.)

   ii. Compute the marginal revenue product of labor as a function of L.

   iii. Compute the profit-maximizing choice of labor L^*. Then compute the price p^*.

c. (3 pts.) In the long run, Mishka's chooses both L and K. Suppose Mishka's wants to produce $q = 20$. If $w = 10$ and $r = 40$, what is the cheapest combination of labor (L^*) and capital (K^*) it can use? (Hint: start by writing the cost-minimization problem.)