Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #3

Due: Friday, October 18th at 5:00pm

1 Expensive clients (12 pts.)

A barber shop offers haircuts to both students and faculty. Student demand for haircuts is given by $p_S(Q_S) = 24 - \frac{1}{4}Q_S$. Faculty demand for haircuts is given by $p_F(Q_F) = 24 - \frac{1}{2}Q_F$. Students have more hair than professors (even the young professors), and longer hair costs more to cut. Reflecting this fact, the barber shop's total costs are

$$C(Q_S, Q_F) = 16Q_S + 10Q_F$$

Suppose first that the barber shop can engage in perfect price discrimination.

a. (3 pts.) How many students get haircuts (Q_S^*)? How many faculty get haircuts (Q_F^*)? How much profit will the barber shop make?

The barber shop sells haircuts to everyone whose willingness to pay exceeds the marginal cost of selling them a haircut. For students, the quantity is sold is determined by the equation

$$p_S(Q_S^*) = \frac{\partial}{\partial Q_S} C(Q_S, Q_F) \implies 24 - \frac{1}{4}Q_S^* = 16 \implies Q_S^* = 32$$

For faculty, the quantity is determined by

$$p_F(Q_F^*) = \frac{\partial}{\partial Q_F} C(Q_S, Q_F) \implies 24 - \frac{1}{2}Q_F^* = 10 \implies Q_F^* = 28$$

We can calculate the total profit by adding the profits from student haircuts plus the profits from faculty haircuts. The profit from students is the area below the student demand curve and above the marginal cost, which equals $\frac{1}{2}(32-0)(24-16) = 128$. The profit from faculty is the area below the faculty demand curve and above the marginal cost, which equals $\frac{1}{2}(28-0)(24-10) = 196$. Total profits are therefore $128+196 = 324$.

b. (3 pts.) Under perfect price discrimination, is each of these statements true or false? Briefly explain your reasoning.

i. Every faculty member with positive willingness to pay ends up getting a haircut.
ii. Among the people who get haircuts, students pay more than faculty on average.
iii. The cheapest haircut sold is sold to a faculty member

i. is false: only faculty members with willingness to pay of at least 10 get haircuts. ii. is true: the average price is \$20 for the students and \$17 for the faculty members. iii. is true: the least any student pays for a haircut is \$16, whereas one of the professors gets a haircut for just \$10.

Now suppose that the barbershop cannot engage in personalized pricing. However, it is able to offer one price for students and a different price for faculty.

c. (3 pts.) Find the monopoly's profit-maximizing prices p_S^* and p_F^* under group price discrimination. Which group is charged a bigger price markup?

The monopoly solves

$$\max_{Q_S,Q_F} \pi(Q_S,Q_F) = (24 - \frac{1}{4}Q_S)Q_S + (24 - \frac{1}{2}Q_F)Q_F - 16Q_S - 10Q_F$$

The FOC for students is

$$24 - \frac{1}{2}Q_S - 16 = 0 \implies Q_S^* = 16 \implies p_S^* = 20$$

The FOC for faculty is

$$24 - Q_F - 10 = 0 \implies Q_F^* = 14 \implies p_F^* = 17$$

So students are charged higher prices. A bit surprisingly, however, they are actually charged lower markups: the student markup is $\frac{p-MC}{MC} = \frac{20-16}{16} = 25\%$, whereas the faculty markup is $\frac{p-MC}{MC} = \frac{17-10}{10} = 70\%$.

Upset about discriminatory prices, student groups organize protests against the barber shop, using the catchy slogan "It's unfair / to tax our hair!" The protests go viral, and the barber shop reluctantly agrees to charge everybody the same price, regardless of cost.

d. (3 pts.) Compute the market demand curve $Q(p)$, then write the barber shop's profits as a function of p. (Be careful with the costs!) What price will the barber shop charge?

Inverting each group's demand curve gives us $Q_S(p) = 96 - 4p$ and $Q_F(p) = 48 - 2p$, so that total demand is $Q(p) = 144 - 6p$. Profits are therefore

$$\pi = pQ(p) - 16Q_S(p) - 10Q_F(p) = p(144 - 6p) - 16(96 - 4p) - 10(48 - 2p)$$

That's pretty ugly, so let's simply it before we take any derivatives:

$$\pi = 228p - 6p^2 - 1536$$

The FOC gives $228 - 12p = 0 \implies p^* = 19$. This seems reasonable: forced to choose a single price, the barber shop chooses one that's in between its preferred student price and its preferred faculty price.

2 Expensive tastes (9 pts.)

FancyPants Vineyard sells bottles of wine both to tourists who come for wine tastings and to foreign wholesalers. Demand from tourists is given by

$$p_T(Q_T) = 48 - Q_T$$

Wholesale demand is perfectly elastic at price $p_W = 32$. Fancypants's total costs are

$$C(Q_T, Q_W) = (Q_T + Q_W)^2$$

where Q_T and Q_W are the quantities sold to tourists and wholesalers, respectively.

a. (3 pts.) Write FancyPants's profit function in terms of Q_T and Q_W. What is its total tourist revenue, expressed as a function of Q_T? What is its total wholesaler revenue, expressed as a function of Q_W?

   Profits are given by: $\pi(Q_T, Q_W) = (48 - Q_T)\, Q_T + 32Q_W - (Q_T + Q_W)^2$. Total tourist revenue is $R(Q_T) = (48 - Q_T)Q_T$. Total wholesaler revenue is $R(Q_W) = 32Q_W$.

b. (3 pts.) Solve for FancyPants's optimal quantities Q_T^* and Q_W^*.

   We have two choice variables, so we need two FOCS. For Q_T, the FOC is:

   $$48 - 2Q_T = 2(Q_T + Q_W)$$

   For Q_W, the FOC is

   $$32 = 2(Q_T + Q_W)$$

   Together, these equations imply that $48 - 2Q_T = 32 \implies Q_T^* = 8 \implies Q_W^* = 8$.

c. (3 pts.) A trade war disrupts FancyPants's access to foreign markets, so that it has to choose $Q_W^* = 0$. Will Q_T^* increase, decrease, or stay the same? What about p_T^*? What about FancyPants's profits?

   Q_T^* goes up, p_T^* goes down, and FancyPants's profits fall. Mathematically: Q_T^* rises from 8 to 12, p_T^* falls from 40 to 36, and profits fall from 320 to 288. Intuitively: if Q_T^* stayed the same, the firm's marginal revenue would exceed its marginal cost, so it will increase Q_T, which depresses the price. Since the firm could have chosen $Q_W = 0$ before, a revealed preference argument tells us that profits must fall.

3 Expensive coffee (9 pts.)

Mishka's Cafe produces coffee using a mixture of labor and capital, with the production function $q(L, K) = 10\sqrt{LK}$. In the short run, however, its capital stock is fixed at the level $\overline{K} = 4$, so that Mishka's has a short-run production function $q(L) = 20\sqrt{L}$.

a. (3 pts.) Suppose that Mishka's is both a price-taker and a wage-taker, facing an output price $p = 3$ and a wage rate $w = 10$.

   i. Compute the marginal physical product of labor as a function of L.

   ii. Compute the marginal revenue product of labor as a function of L.

   iii. Compute the profit-maximizing choice of labor L^*.

The MPPL is given by $q'(L) = \frac{10}{\sqrt{L}}$. The MRPL is given by $pq'(L) = \frac{30}{\sqrt{L}}$. To find L^*, we solve the profit-maximization problem

$$\max_L \ \pi(L) = pq(L) - wL = (3)(20\sqrt{L}) - (10)(L) = 60\sqrt{L} - 10L$$

Taking the FOC, or (equivalently) setting the MRPL $= w$, gives

$$\frac{30}{\sqrt{L}} = 10 \implies \sqrt{L} = 3 \implies L^* = 9$$

b. (3 pts.) Mishka's is still a wage-taker (with $w = 10$), but now suppose that it faces downward sloping demand for coffee, given by $p(q) = 10 - \frac{1}{10}q$.

   i. Write the profit as a function of L. (No "q" terms should appear in your answer.)

   ii. Compute the marginal revenue product of labor as a function of L.

   iii. Compute the profit-maximizing choice of labor L^*. Then compute the price p^*.

If Mishka's hires L workers, its output is $q(L) = 20\sqrt{L}$, which means that its price is $p(q(L)) = 10 - \frac{1}{10}q(L) = 10 - 2\sqrt{L}$. That means its total revenue is

$$R(q(L)) = p(q(L))q(L) = (10 - 2\sqrt{L}) \cdot 20\sqrt{L} = 200\sqrt{L} - 40L$$

and its profit is

$$\pi = R(q(L)) - wL = 200\sqrt{L} - 40L - 10L = 200\sqrt{L} - 50L$$

The marginal revenue product of labor is $MRPL = \frac{100}{\sqrt{L}} - 40$.

We can find the profit-maximizing choice of labor by setting $MRPL = w$:

$$\frac{100}{\sqrt{L}} - 40 = 10 \implies L^* = 4$$

The quantity produced is $q^* = 20\sqrt{4} = 40$, which implies that the price is $p^* = 10 - \frac{1}{10}40 = 6$.

c. (3 pts.) In the long run, Mishka's chooses both L and K. Suppose Mishka's wants to produce $q = 20$. If $w = 10$ and $r = 40$, what is the cheapest combination of labor (L^*) and capital (K^*) it can use? (Hint: start by writing the cost-minimization problem.)

Mishka solves the cost-minimization problem

$$\min_{L,K} \ wL + rK \quad \text{subject to the constraint} \quad q(L,K) = 20$$

which simplifies to

$$\min_{L,K} \ 10L + 40K \quad \text{subject to the constraint} \quad 10\sqrt{LK} = 20$$

We can rewrite the constraint as $K = \frac{4}{L}$. Substituting this into the objective function:

$$\min_{L} \ 10L + 40\frac{4}{L} = 10L + \frac{160}{L}$$

Since $\frac{d}{dL}\left(\frac{1}{L}\right) = \frac{d}{dL}(L^{-1}) = -L^{-2} = -\frac{1}{L^2}$, the FOC is

$$10 - \frac{160}{L^2} = 0 \implies L^* = 4 \implies K^* = 1$$