Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #4

Due: Saturday, November 2nd at 5:00pm

1 Static games (9 pts.)

For each of the following games:

- Circle all payoffs corresponding to a player's best response.
- Identify any/all strictly dominant strategies (or indicate that there are none).
- Identify any/all strictly dominated strategies (or indicate that there are none).
- Identify any/all pure strategy Nash equilibria by writing the equilibrium strategies as an ordered pair. (If there is no PSNE, write "no PSNE".)

a. (3 pts.) Two friends are deciding what costumes to wear for Halloween.

		Matt: Parrot	Matt: Zombie
Grace	Vampire	6, 1	10, 7
	Pirate	8, 4	10, 3

There are no strictly dominant strategies (though Grace's strategy Pirate is weakly dominant). There are no strictly dominated strategies. There are two PSNEs: (Pirate, Parrot) and (Vampire, Zombie).

b. (3 pts.) It's the end of the soccer game, and it's time for a penalty shot.

		Goalie: Left	Goalie: Right
Kicker	Left	0, 2	2, 0
	Right	2, 0	0, 2
	Oops	−3, 1	−3, 1

There are no strictly dominant strategies, but Kicker's strategy Oops is strictly dominated by either Left or Right (since either of these strategies would give Kicker a higher payoff than Oops under any circumstances). There is no PSNE. (The absence of a PSNE here is typical of games in which one player is trying to "outguess" the other one. In "outguessing games", once each player sees what the other one chose, one of them will realize that she guessed wrong—i.e., that she didn't play a best response—and that player will regret her choice.)

c. (3 pts.) Two doctors are deciding whether to order pizza for lunch.

		Dr. Blah	
		Whatevs	¯_(ツ)_/¯
Dr. Meh	Sure	2, 1	2, 1
	Okay	2, 1	2, 1

As their names imply, Dr. Meh and Dr. Blah are completely indifferent. None of the strategies are strictly dominant, none of them are strictly dominated, every action is a best response, and every combination of strategies is a PSNE: (Sure, Whatevs), (Sure, ¯_(ツ)_/¯), (Okay, Whatevs), and (Okay, ¯_(ツ)_/¯).

2 True or false (6 pts.)

Indicate whether each of the following statements is true or false. You do not need to provide an explanation this time—just true or false.

a. (1 pt.) Every static game has at least one pure strategy Nash equilibrium.

False. Every static game has at least one *mixed* strategy Nash equilibrium, but we've seen games that have no pure strategy Nash equilibrium: examples include "Name the Biggest Number" and "Rock, Paper, Scissors".

b. (1 pt.) If a strategy is weakly dominant, then it is a best response against any strategy chosen by the other player.

True. A weakly dominant strategy never yields a lower payoff than any alternative strategy, so it is always a best response (though not necessarily the *only* best response.)

c. (1 pt.) In a dynamic game, it is always better to go first.

False. In class, we've discussed games that exhibit "second mover advantage": for example, in the dynamic version of "Name the Biggest Number", it's better to go second, since the second mover can always find a number bigger than the number chosen by the first mover.

d. (1 pt.) Every pure strategy Nash equilibrium is also a dominant strategy solution.

False. For example, in the Cuban Missile Crisis game, "(Attack, Attack)" is a PSNE but not a dominant strategy solution. (However, the converse statement is true: every dominant strategy solution is a PSNE, since a strictly dominant strategy is always a best response.)

e. (1 pt.) If a player has a strictly dominant strategy, then all of her other strategies must be strictly dominated.

True. If strategy "A" is strictly dominant, then any other strategy "B" is strictly dominated by A, since playing B always yields a lower payoff than playing A.

f. (1 pt.) When deciding which action to take, each player tries to maximize the sum of the players' equilibrium payoffs.

False. Each player just cares about his own payoff, not about the sum of the two payoffs.

3 Coordinated care (9 pts.)

California and Nevada are adopting electronic medical records (EMR) systems, provided by either Kareo or Praxis. Each state has a preference for using one of these systems rather than the other, but all else equal, both states would like to use the same system since doing so makes it easier to coordinate healthcare for people who commute across state lines.

		Nevada	
		Kareo	Praxis
California	Kareo	6, 1	0, 0
	Praxis	0, 0	3, 5

a. (1 pt.) What is California's best response against Nevada choosing Praxis?

California's best response is to choose Praxis as well.

b. (2 pts.) Identify any/all pure strategy Nash equilibria (in each case, list the strategies as an ordered pair). For each PSNE, also indicate the corresponding equilibrium payoffs.

There are two PSNEs: (Kareo, Kareo) and (Praxis, Praxis). Each state prefers its favorite system, but it prioritizes coordinating with its neighbor over adopting its preferred system.

c. (3 pts.) Suppose that California picks Kareo with probability $\frac{3}{4}$ and that Nevada picks Kareo with probability $\frac{1}{3}$. Show that this cannot be a mixed strategy Nash equilibrium. Which player is not playing a best response?

Given Nevada's strategy, California's expected payoffs from each of its options are as follows:

$$\text{expected payoff from Kareo} = \frac{1}{3} \times 6 + \frac{2}{3} \times 0 = 2$$
$$\text{expected payoff from Praxis} = \frac{1}{3} \times 0 + \frac{2}{3} \times 3 = 2$$

Since both strategies yield the same expected profit, California is indifferent between the two strategies, which implies that any mixed strategy is a best response. So California is playing a best response.

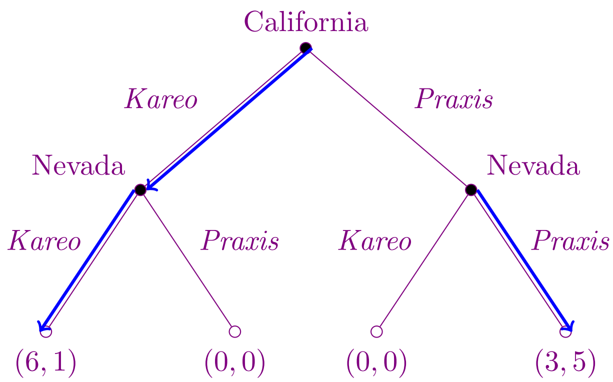
Now let's consider Nevada. Given California's strategy, Nevada's expected payoffs from each of its options are as follows:

$$\text{expected payoff from Kareo} = \frac{3}{4} \times 1 + \frac{1}{4} \times 0 = \frac{3}{4}$$
$$\text{expected payoff from Praxis} = \frac{3}{4} \times 0 + \frac{1}{4} \times 5 = \frac{5}{4}$$

Since Praxis yields a bigger expected payoff, Nevada can increase its expected payoff by playing Praxis for sure. Therefore, Nevada was not playing a best response in the first place. Since not all players were playing a best response, the original pair of strategies cannot be a mixed strategy Nash equilibrium.

d. (3 pts.) Now imagine that California makes its decision first. California's decision will be public knowledge, so Nevada will get to see what California did before making its decision. Draw the game tree corresponding to this dynamic game. What will California do? How will Nevada respond? What are the equilibrium payoffs?

Here's the game tree, with an arrow showing each player's best response at each of its decision nodes:



Whatever California picks, Nevada will respond by adopting the same system. California understands this, so it should pick the system it prefers, which is Kareo: Nevada will copy California's decision, and California will get its preferred outcome. The equilibrium payoffs are (6,1).

4 Bad intersection (6 pts.)

"You're a rotten driver," I protested. "Either you ought to be more careful or you oughtn't to drive at all."
"I am careful."
"No, you're not."
"Well, other people are," she said lightly.
"What's that got to do with it?"
"They'll keep out of my way," she insisted. "It takes two to make an accident."
—F. Scott Fitzgerald, The Great Gatsby

Two drivers, Jordan Baker and Nick Carraway, are approaching each other on a dark road. In the quotation above, Jordan is the "rotten driver".

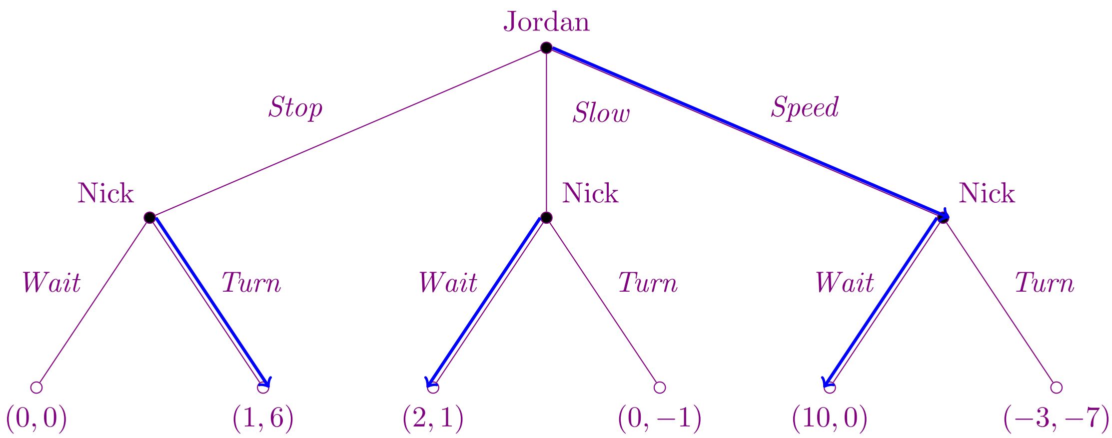
		Nick	
		Wait	Turn
Jordan	Stop	$0, 0$	$1, 6$
	Slow	$2, 1$	$0, -1$
	Speed	$10, 0$	$-3, -7$

a. (1 pt.) Suppose this is a static game, where both players move at the same time. Circle all payoffs corresponding to best responses. Find any/all pure strategy Nash equilibria.

See circled payoffs above. The two PSNEs are (Stop, Turn) and (Speed, Wait).

b. (3 pts.) Now suppose Jordan moves first, followed by Nick. (Her car is moving faster, so if she wants to brake, she has to react quickly.) Draw the game tree. Then use backward induction to find the subgame perfect Nash equilibrium. (Remember to say what each player chooses at every decision node.) What are the equilibrium payoffs?

Here's the game tree:



The SPNE is: "Jordan plays Speed. If Jordan plays Stop, Nick plays Turn. If Jordan plays Slow or Speed, Nick plays Wait." The equilibrium payoffs are (10,0).

c. (2 pt.) Now suppose that Nick moves first, followed by Jordan. Assuming that both players are rational, which action will Nick choose, and how will Jordan respond? Does this game exhibit first-mover advantage or second-mover advantage?

If Nick chooses Wait, then Jordan will pick Speed and Nick will get a payoff of 0. If Nick chooses Turn, then Jordan will pick Stop and Nick will get a payoff of 6. Since $6 > 0$, Nick will choose Turn and Jordan will respond by playing Stop. This game has first-mover advantage: each player gets a higher payoff if he/she moves first than if he/she moves second.

