Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

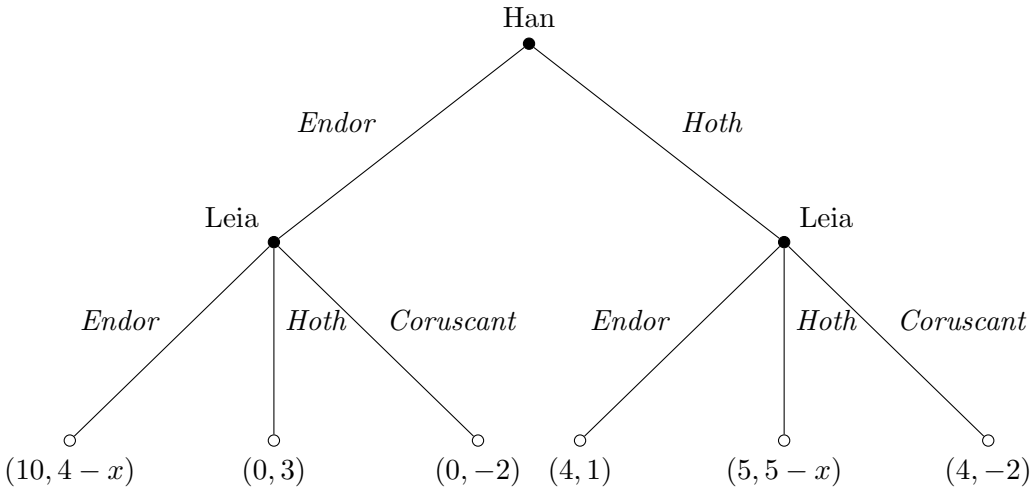
Homework #5

Due: Saturday, November 9nd at 5:00pm

# 1 Star Wars (9 pts. total)

Han and Leia are deciding where to go on vacation—Endor for hiking or Hoth for skiing. (Leia can also choose to go on a business trip to Coruscant, but Han never considers going there: he's been working too hard and wants to relax.) Since they're traveling from different planets, Han picks his destination first, and Leia decides whether to join him or avoid him.

The game tree is as follows. (Han is player 1, so his payoffs are written first.) The payoffs marked "$x$" will change throughout the problem, depending on whether Leia is mad at Han.




a. (1 pt.) How many *decision nodes* does this game have? How many *subgames*?

b. (3 pts.) Suppose $x = 0$. Use backward induction to find the subgame perfect Nash equilibrium (there's only one). Remember to describe each player's complete "if-then" strategy. What are the equilibrium payoffs? Which planet does Leia end up visiting?

c. (3 pts.) Now suppose $x = 5$. (Han and Leia recently had an argument, and Leia doesn't really want to be around him.) Find the SPNE. What are the equilibrium payoffs?

d. (2 pts.) Now suppose $x = 4$. (Han has apologized, and Leia is starting to forgive him.) Now there are *two* subgame perfect Nash equilibria in which Han and Leia both use pure strategies. (Ignore any non-pure mixed strategies.) Find both of these SPNEs.

## 2 Second breakfast (6 pts. total)

Two hungry hobbits are deciding what to have for second breakfast this morning. Frodo is bringing the food, and Samwise is bringing something to drink.

| | | Samwise | | |
|---|---|---|---|---|
| | | Tea | Cider | Mead |
| Frodo | Mutton | $0, 1$ | $2, 3$ | $4, 2$ |
| | Lembas | $1, 2$ | $2, -1$ | $4, 3$ |
| | Cake | $3, 7$ | $10, 5$ | $1, 1$ |

a. (3 pts.) Suppose this is a static game. Circle all payoffs corresponding to a player's best response, then list any/all pure strategy Nash equilibria. (If there aren't any, write "no pure strategy Nash equilibria".) Be sure to write the *strategies*, not the payoffs.

b. (3 pts.) Now suppose that Frodo moves first, so that this is a dynamic game. Draw the game tree, then (as we did in Lecture Note 9) draw shaded lines to indicate the action chosen at each decision node. Find the subgame-perfect Nash equilibrium. (Again, remember to describe each player's complete "if-then" strategy). What will the hobbits have for second breakfast? What are the equilibrium payoffs?

## 3 Pain relievers (9 pts. total)

Tylenol and Advil are two of the world's leading pain medications. Since both drugs have already been developed, and since drug production typically involves very low marginal costs, we will start by assuming that the cost of production is zero for each firm.

Suppose that Tylenol and Advil are identical products, so that there is a single demand curve for pain-relief medications. Letting $q_T$ and $q_A$ denote the quantity of each drug,

$$p(Q) = 36 - Q \qquad \text{where} \qquad Q = q_T + q_A$$

Tylenol and Advil compete as in Cournot, by choosing their quantities at the same time.

a. (3 pts.) Write Tylenol's profit-maximization problem. Find Tylenol's best-response function $BR_T(\widehat{q}_A)$, where $\widehat{q}_A$ is Tylenol's "guess" about what Advil will produce.

b. (3 pts.) Find the Nash equilibrium quantities $q_T^*$ and $q_A^*$. Then find the equilibrium price $p^*$. Compute Tylenol's profit.

c. (3 pts.) Now suppose Advil's cost function is $C(q_A) = 48q_A$. (Tylenol still has zero costs.) Find the new Nash equilibrium quantities $q_T^*$ and $q_A^*$. Relative to part b, how has the increase in Advil's cost of production affected Tylenol's profit?

# 4 Electric cars (6 pts. total)

Tesla was an early entrant into the market for electric cars, and other automobile companies have been trying to catch up.

Suppose that Tesla and Honda are each deciding how many electric cars to produce. They compete as in the Stackelberg model, with Tesla moving first and Honda moving second. The market demand curve is

$$p(Q) = 30 - Q \qquad \text{where} \qquad Q = q_T + q_H$$

where $q_T$ is Tesla's quantity produced and $q_H$ is Honda's quantity produced. Each firm has a constant marginal cost of 10. There are no fixed costs.

a. (3 pts.) Since Honda moves second, backwards induction tells us that we should start by figuring out Honda's best response to Tesla's choice of output. Write Honda's profits in terms of $q_T$ and $q_H$. Then find Honda's best-response function, indicating Honda's choice $q_H^*$ as a function of Tesla's choice $q_T$.

b. (3 pts.) Once we know Honda's best-response function, we can use that information to figure out Tesla's optimal choice of output. Write Tesla's profit-maximization problem in terms of $q_T$ (by using Honda's best-response function to express Honda's choice as a function of $q_T$). Find the Nash equilibrium quantities $q_T^*$ and $q_H^*$ in this Stackelberg game.