Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

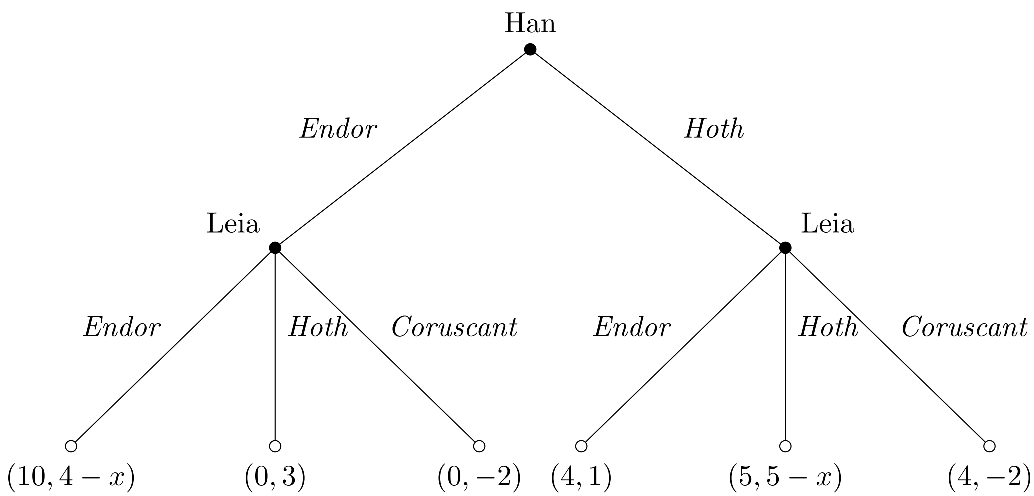
Homework #5

Due: Saturday, November 9nd at 5:00pm

1 Star Wars (9 pts. total)

Han and Leia are deciding where to go on vacation—Endor for hiking or Hoth for skiing. (Leia can also choose to go on a business trip to Coruscant, but Han never considers going there: he's been working too hard and wants to relax.) Since they're traveling from different planets, Han picks his destination first, and Leia decides whether to join him or avoid him.

The game tree is as follows. (Han is player 1, so his payoffs are written first.) The payoffs marked "x" will change throughout the problem, depending on whether Leia is mad at Han.



a. (1 pt.) How many *decision nodes* does this game have? How many *subgames*?

There are three decision nodes (the black circles where a player is making a decision), and there are three subgames (one subgame starting at each decision node). Remember that the game as a whole is considered a subgame of itself.

b. (3 pts.) Suppose $x = 0$. Use backward induction to find the subgame perfect Nash equilibrium (there's only one). Remember to describe each player's complete "if-then" strategy. What are the equilibrium payoffs? Which planet does Leia end up visiting?

Han's strategy: "Endor". Leia's strategy: "If Han chooses Endor, I will choose Endor. If Han chooses Hoth, I will choose Hoth." (The story here is that, when $x = 0$, Han and Leia are getting along well, so she wants to join him on vacation.) The equilibrium payoffs are (10, 4). If the players use these strategies, then Han will play the action Endor, and Leia will respond by picking the action Endor as well.

c. (3 pts.) Now suppose $x = 5$. (Han and Leia recently had an argument, and Leia doesn't really want to be around him.) Find the SPNE. What are the equilibrium payoffs?

Han's strategy: "Hoth". Leia's strategy: "If Han chooses Endor, I will choose Hoth. If Han chooses Hoth, I will choose Endor." (The story here is that, when $x = 5$, Leia is actively trying to avoid Han. Han recognizes that he is going to be alone either way, and given his payoffs, he would rather be skiing by himself than hiking by himself.) Under these strategies, Han will end up at Hoth and Leia will end up at Endor, so the equilibrium payoffs are (4, 1).

d. (2 pts.) Now suppose $x = 4$. (Han has apologized, and Leia is starting to forgive him.) Now there are *two* subgame perfect Nash equilibria in which Han and Leia both use pure strategies. (Ignore any non-pure mixed strategies.) Find both of these SPNEs.

If Han chooses Endor, Leia still prefers to go to Hoth, so Han will get a payoff of 0. If he chooses Hoth, Leia is indifferent between going to Endor (in which case Han gets a payoff of 4) and going to Hoth (in which case Han gets a payoff of 5).

Either way, Han will get a higher payoff from choosing Hoth than from choosing Endor, so Han will choose Hoth no matter what, but Leia's indifference results in two (pure strategy) subgame-perfect Nash equilibria:

- First SPNE: Han's strategy: "Hoth." Leia's strategy: "If Han chooses Endor, I will choose Hoth. If Han chooses Hoth, I will choose Endor."
- Second SPNE: Han's strategy: "Hoth." Leia's strategy: "If Han chooses Endor, I will choose Hoth. If Han chooses Hoth, I will choose Hoth."

(In case you're wondering, there are also additional *mixed strategy* subgame-perfect Nash equilibria in which Han's strategy is "Hoth" and Leia's strategy is: "If Han chooses Endor, I will choose Hoth. If Han chooses Hoth, I will play a mixed strategy, going to Endor with probability p and going to Hoth with probability $1 - p$." There are infinitely many such mixed-strategy Nash equilibria—one for every possible value of p between 0 and 1—all of these these equilibria are subgame-perfect.)

2 Second breakfast (6 pts. total)

Two hungry hobbits are deciding what to have for second breakfast this morning. Frodo is bringing the food, and Samwise is bringing something to drink.

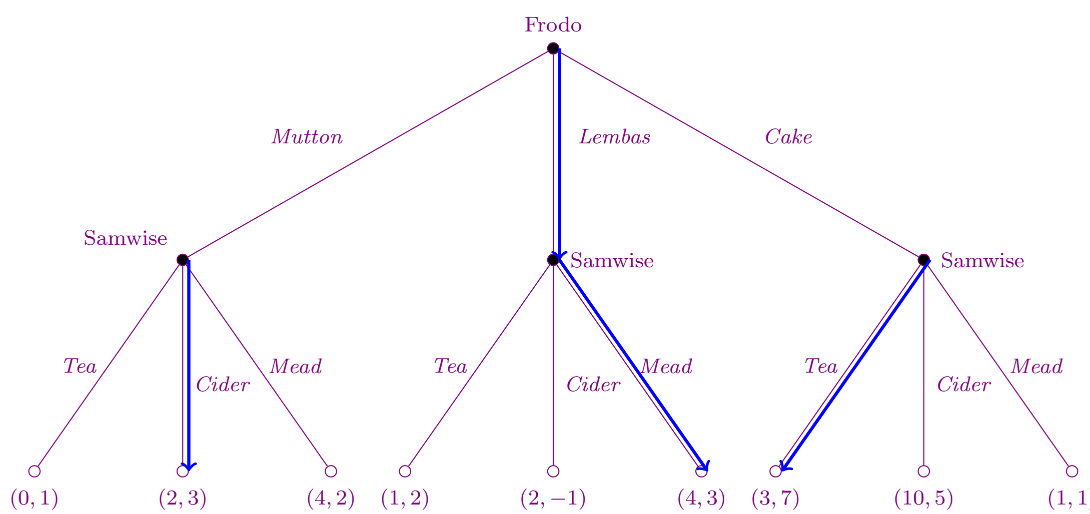
		Samwise		
		Tea	Cider	Mead
Frodo	Mutton	0, 1	2, (3)	(4), 2
	Lembas	1, 2	2, −1	(4), (3)
	Cake	(3), (7)	(10), 5	1, 1

a. (3 pts.) Suppose this is a static game. Circle all payoffs corresponding to a player's best response, then list any/all pure strategy Nash equilibria. (If there aren't any, write "no pure strategy Nash equilibria".) Be sure to write the *strategies*, not the payoffs.

See the circled payoffs in the payoff matrix above: to make things clearer, I used blue for Frodo's payoffs and red for Samwise's payoffs. There are two pure-strategy Nash equilibria: (Lembas, Mead) and (Cake, Tea). These are the cells where each player is playing a best response against the opposing strategy, so that both payoffs are circled.

b. (3 pts.) Now suppose that Frodo moves first, so that this is a dynamic game. Draw the game tree, then (as we did in Lecture Note 9) draw shaded lines to indicate the action chosen at each decision node. Find the subgame-perfect Nash equilibrium. (Again, remember to describe each player's complete "if-then" strategy). What will the hobbits have for second breakfast? What are the equilibrium payoffs?

Here's the game tree:



The subgame-perfect Nash equilibrium is as follows. Frodo's strategy: "Lembas". Samwise's strategy: "If Frodo chooses Mutton, I will choose Cider. If Frodo chooses Lembas, I will choose Mead. If Frodo chooses Cake, I will choose Tea." Given these strategies, second breakfast will be Lembas and Mead, with equilibrium payoffs (4, 3).

3 Pain relievers (9 pts. total)

Tylenol and Advil are two of the world's leading pain medications. Since both drugs have already been developed, and since drug production typically involves very low marginal costs, we will start by assuming that the cost of production is zero for each firm.

Suppose that Tylenol and Advil are identical products, so that there is a single demand curve for pain-relief medications. Letting q_T and q_A denote the quantity of each drug,

$$p(Q) = 36 - Q \qquad \text{where} \qquad Q = q_T + q_A$$

Tylenol and Advil compete as in Cournot, by choosing their quantities at the same time.

a. (3 pts.) Write Tylenol's profit-maximization problem. Find Tylenol's best-response function $BR_T(\widehat{q}_A)$, where $\widehat{q}_A$ is Tylenol's "guess" about what Advil will produce.

Tylenol's profit-maximization problem is

$$\max_{q_T} \pi = (36 - q_T - \widehat{q}_A)q_T$$

Partially differentiating profits with respect to q_T gives the FOC

$$36 - 2q_T - \widehat{q}_A = 0 \implies q_T^* = BR_T(\widehat{q}_A) = 18 - \frac{1}{2}\widehat{q}_A$$

b. (3 pts.) Find the Nash equilibrium quantities q_T^* and q_A^*. Then find the equilibrium price p^*. Compute Tylenol's profit.

This is a symmetric problem, so Advil's best-response function is

$$q_A^* = BR_A(\widehat{q}_T) = 18 - \frac{1}{2}\widehat{q}_T$$

Combining these best-response functions, and setting $\widehat{q}_T = q_T^*$ and $\widehat{q}_A = q_A^*$, we get a system of two equations in two unknowns:

$$q_T^* = 18 - \frac{1}{2}q_A^*$$
$$q_A^* = 18 - \frac{1}{2}q_T^*$$

Solving this system of equations yields $q_T^* = q_A^* = p^* = 12$. Each firm sells 12 units at a price of 12, so that its total revenue is $12 \times 12 = 144$. Since there are no costs of production, each firm's profit is $144 - 0 = 144$.

c. (3 pts.) Now suppose Advil's cost function is $C(q_A) = 48q_A$. (Tylenol still has zero costs.) Find the new Nash equilibrium quantities q_T^* and q_A^*. Relative to part b, how has the increase in Advil's cost of production affected Tylenol's profit?

The quick way to solve this problem is to notice that Advil's marginal cost (48) always exceeds the market price (which can never exceed 36). Therefore, Advil will choose not to operate, setting $q_A^* = 0$. (In fact, this is Advil's strictly dominant strategy.) Tylenol's best response against Advil's strategy is to choose

$$q_T^* = BR_T(0) = 18 - \frac{1}{2} \cdot 0 = 18$$

So the Nash equilibrium is $q_T^* = 18$, $q_A^* = 0$. The price is $p^* = 36 - 18 - 0 = 18$, so Tylenol's profits are $p^* q_T^* - C(q_T^*) = 18 \times 18 - 0 = 324$. Since $324 > 144$, Tylenol is making more profit in this equilibrium than it did when Advil was also choosing to produce a positive quantity. (Since Advil is not producing anything, Tylenol is actually producing the optimal quantity for a uniform-pricing monopolist.)

A longer way to solve this problem is to start by looking for an interior solution. Tylenol's profit-maximization problem (for a given "guess" about Advil's output) is unchanged, so its best-response function is also unchanged:

$$q_T^* = BR_T(\widehat{q}_A) = 18 - \frac{1}{2}\widehat{q}_A$$

But Advil's problem-maximization problem has changed because it now has costs:

$$\max_{q_A} \, (36 - \widehat{q}_T - q_A)q_A - 48q_A$$

Taking the FOC and solving for q_A yields Advil's best-response function:

$$q_A^* = BR_A(\widehat{q}_T) = -6 - \frac{1}{2}\widehat{q}_T$$

If we combine the best-response functions and solve the system of equations, we get: $q_T^* = 28$ and $q_A^* = -20$. But that's no good: since quantities cannot be negative, this is an invalid solution. That means there is no interior solution.

That means we have a corner solution, with Advil choosing $q_A^* = 0$. But remember: even though $q_T^* = 28$ looks like a reasonable solution, we obtained it under the assumption that there's an interior solution for both firms. Since that assumption has turned out to be false, we have to redo Tylenol's problem with q_A set to zero (again, Advil is effectively a uniform-pricing monopolist now). In this case, we don't have to go all the way back to Advil's profit-maximization problem: we can just plug $\widehat{q}_A = 0$ into Tylenol's best-response function:

$$q_T^* = BR_T(0) = 18 - \frac{1}{2} \cdot 0 = 18$$

So, as I argued above, the correct answer is $q_T^* = 18$ and $q_A^* = 0$.

The question didn't ask this, but it's also interesting to examine how this change in Advil's costs has affected total profits in this sector. In part b, total profits were $144 + 144 = 288$. Now, total profits have risen to $324 + 0 = 324$.

What's going on here? Tylenol's withdrawal from the market has turned a duopoly into a monopoly: since there is less competition, the equilibrium price is higher than it was before, and the market has become more profitable for firms as a whole (while also being worse for consumers, who face a higher price).

4 Electric cars (6 pts. total)

Tesla was an early entrant into the market for electric cars, and other automobile companies have been trying to catch up.

Suppose that Tesla and Honda are each deciding how many electric cars to produce. They compete as in the Stackelberg model, with Tesla moving first and Honda moving second. The market demand curve is

$$p(Q) = 30 - Q \qquad \text{where} \qquad Q = q_T + q_H$$

where q_T is Tesla's quantity produced and q_H is Honda's quantity produced. Each firm has a constant marginal cost of 10. There are no fixed costs.

a. (3 pts.) Since Honda moves second, backwards induction tells us that we should start by figuring out Honda's best response to Tesla's choice of output. Write Honda's profits in terms of q_T and q_H. Then find Honda's best-response function, indicating Honda's choice q_H^* as a function of Tesla's choice q_T.

Honda's total cost of production is $C(q_H) = 10q_H$, which means that its profits are $\pi = (30 - q_T - q_H)q_H - 10q_H$. Its profit-maximization problem is therefore

$$\max_{q_H} \ \pi = (30 - q_T - q_H)q_H - 10q_H$$

(I've written Tesla's quantity as q_T rather than $\widehat{q}_T$ because Honda gets to see exactly what Tesla chose, rather than having to make an educated guess about Tesla's choice.)

Partially differentiating profits with respect to q_H, and setting the marginal profit equal to zero, we get the FOC

$$30 - q_T - 2q_H^* - 10 = 0$$

Solving for q_H yields the best-response function:

$$q_H^* = BR_H(q_T) = 10 - \frac{1}{2}q_T$$

b. (3 pts.) Once we know Honda's best-response function, we can use that information to figure out Tesla's optimal choice of output. Write Tesla's profit-maximization problem in terms of q_T (by using Honda's best-response function to express Honda's choice as a function of q_T). Find the Nash equilibrium quantities q_T^* and q_H^* in this Stackelberg game.

Plugging Honda's best-response function into the expression for the market price, Tesla's profit-maximization problem is

$$\max_{q_T} \ \pi = \left(30 - q_T - \left(10 - \frac{1}{2}q_T\right)\right) q_T - 10q_T$$

which simplifies to

$$\max_{q_T} \ \pi = 10q_T - \frac{1}{2}q_T^2$$

Taking the FOC yields $10 - q_T^* = 0$, so that $q_T^* = 10$. Plugging this into Honda's best-response function gives us $q_H^* = 5$. Notice that Tesla enjoys first-mover advantage here: even though Tesla and Honda are producing identical products at identical costs, Tesla makes more profit because going first allows it to commit to a high level of output.