Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #6

Due: Saturday, November 23rd at 5:00pm

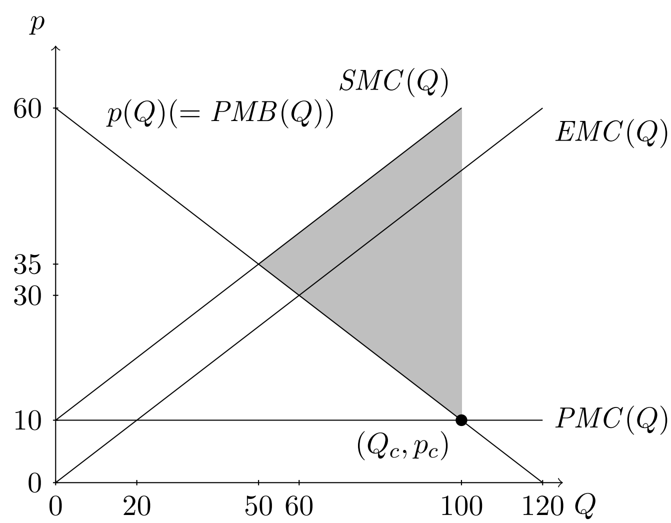
1 Water pollution (12 pt.)

The process of tanning leather creates toxic byproducts that pollute the local water supply. Suppose that a city's tanneries (i.e., producers) compete in a perfectly competitive market, facing demand given by $p(Q) = 60 - \frac{1}{2}Q$ and a constant (private) marginal cost of 10. Leather production imposes external costs equal to $EC(Q) = \frac{1}{4}Q^2$.

a. (3 pt.) Draw a clearly labeled graph representing this market. Include:

- The private marginal benefit curve (i.e., the demand curve)
- The private marginal cost curve (i.e., the supply curve)
- The external marginal cost curve
- The social marginal cost curve

Include axis labels for all points where these curves intersect each other or the axes.



b. (3 pt.) On your graph from part a, mark the competitive quantity (Q_c) and price (p_c). Shade in the deadweight loss and compute its area. Then calculate the consumer surplus, producer surplus, external cost, and (finally) total surplus.

See graph above. We find the competitive equilibrium quantity by setting the demand curve equal to the supply curve:

$$p(Q) = PMC(Q) \implies 60 - \frac{1}{2}Q = 10 \implies Q_c = 100$$

Plugging this quantity into the demand curve gives the equilibrium price $p_c = 10$.

I've shaded the DWL in the graph above: it equals $\frac{1}{2}(100 - 50)(60 - 10) = 1250$. The consumer surplus is the area under the demand curve and above the price: its area is $CS = \frac{1}{2}(100 - 0)(60 - 10) = 2500$. The producer surplus equals zero. The external cost equals $EC(Q) = \frac{1}{4}(100)^2 = 2500$.

Since there is no net tax revenue here (and no external benefit), total surplus is consumer surplus plus producer surplus minus total external cost: $TS = 2500 + 0 - 2500 = 0$.

c. (3 pt.) Find the socially optimal quantity (Q_s). What is the private marginal cost at Q_s? What is the external marginal cost at Q_s? What is the social marginal cost at Q_s?

The social optimum occurs at the point where $SMB(Q_s) = SMC(Q_s)$. Since there is no external benefit, the social marginal benefit equals the private marginal benefit, which is given by the demand curve:

$$SMB(Q_s) = 60 - \frac{1}{2}Q_s = 10 + \frac{1}{2}Q_s \implies Q_s = 50$$

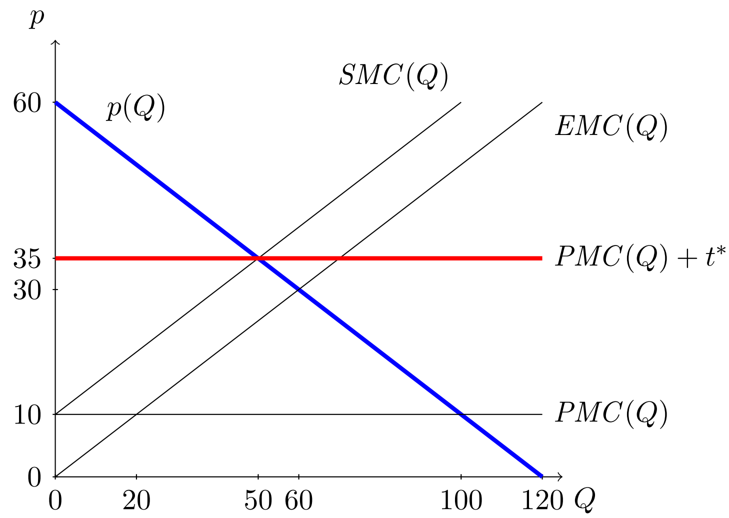
At $Q_s = 50$, the private marginal cost is 10, the external marginal cost is 25, and the social marginal cost is 35.

d. (3 pt.) Suppose that the city imposes a corrective tax t on the tanneries for each unit of output they produce. Find the value t^* that results in the socially optimal amount Q_s being produced. Compute consumer surplus, producer surplus, tax revenue, and external cost under this tax. How does the total surplus compare with the total surplus in part b?

Imposing a specific tax on producers will cause the supply curve to shift up by an amount t. We need to choose t so that the demand curve intersects the new supply curve at Q_s. In other words, we need

$$p(Q_s) = 10 + t^* \implies 60 - \frac{1}{2}(50) = 10 + t^* \implies t^* = 25$$

We can see this solution graphically in the figure below, where I've drawn the new supply curve as a thick red line (and redrawn the demand curve as a thick blue line): these two curves now intersect at the optimum quantity. Note that the optimal corrective tax equals the external marginal cost *evaluated at the social optimum*: $EMC(Q_s) = \frac{1}{2} \times 50 = 25 = t^*$.



Under this optimal corrective tax, the equilibrium price (as paid by consumers) equals 35. We can now compute each component of social welfare under this tax:

- Consumer surplus: $CS = \frac{1}{2}(50 - 0)(60 - 35) = 625$
- Producer surplus $PS = 0$
- Tax revenue $TR = 25 \times 50 = 1250$
- External cost $EC = \frac{1}{4}(50)^2 = 625$

The total surplus equals

$$TS = CS + PS + TR - EC = 625 + 0 + 1250 - 625 = 1250$$

Notice that, as we should expect, the change in total surplus between the competitive equilibrium and the social optimum exactly equals the deadweight loss under the competitive equilibrium.

2 Classifying goods (6 pt.)

a. (2 pt.) Give an example of a public good that we haven't discussed in class. Explain why you consider it a public good.

Many of Facebook's product innovations—such as the "like" button and the newsfeed—can be seen as public goods: they're non-rivalrous because other tech companies can adopt them without affecting Facebook's ability to use its own technology, and they're non-excludable because anyone who uses Facebook's product will learn how they work.

b. (2 pt.) Give an example of a common good that we haven't discussed in class. Explain why you consider it a common good.

The I-80 interstate is a common good: it's rivalrous (the more cars that use it, the more congestion and hence the lower the quality of service provided) and non-excludable (you don't have to pay tolls to use this highway).

c. (2 pt.) Give an example of a club good that we haven't discussed in class. Explain why you consider it a club good.

WGSN is a company that issues trend forecasts for the fashion industry, projecting what kinds of products, colors, etc. will be in vogue in the coming season. WGSN's forecasts are a club good: they are non-rivalrous (sharing the forecasts with one clothing company doesn't interfere with another company's ability to read them), but they are excludable (clothing companies have to subscribe to receive access to WGSN's trend forecasts).

3 Neighborinos (12 pt.)

Homer Simpson and Ned Flanders can both contribute to mowing the grass between their properties. Let Q denote the total number of minutes spent on mowing in a given week, where $Q = q_H + q_F$ is the sum of minutes spent mowing by Homer and by Flanders, respectively.

Homer's demand curve is

$$p_H(Q) = \begin{cases} 50 - Q & \text{for } Q \leq 50 \\ 0 & \text{for } Q > 50 \end{cases}$$

Flanders' demand curve is

$$p_F(Q) = \begin{cases} 10 - \frac{1}{2}Q & \text{for } Q \leq 20 \\ 0 & \text{for } Q > 20 \end{cases}$$

The marginal cost of mowing is the opportunity cost of time, which will change throughout the problem depending on how busy each neighbor is in a given week.

a. (3 pt.) Calculate the social marginal benefit curve as a function of Q. (Hint: it's a "kinked" or piecewise-linear curve, so you'll need two equations to describe it: one for smaller values of Q and one for larger values of Q. You may want to draw a graph.)

We find the social marginal benefit (SMB) curve by summing the two private demand curves vertically. For $Q \leq 20$, both demand curves are positive and the sum is

$$p(Q) = p_H(Q) + p_F(Q) = 50 - Q + 10 - \frac{1}{2}Q = 60 - \frac{3}{2}Q$$

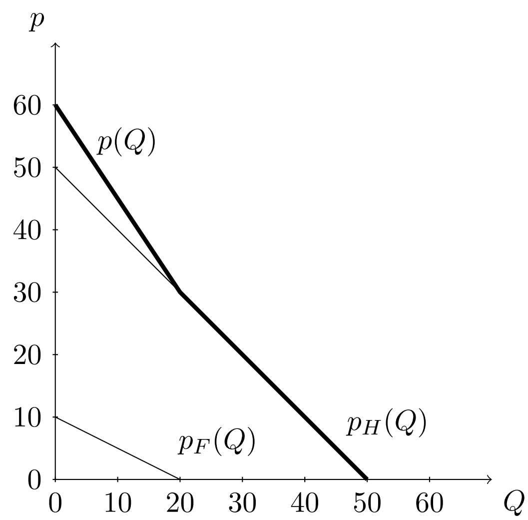
However, for $Q > 20$, Flanders's marginal benefit is zero, so the SMB curve is

$$p(Q) = 50 - Q$$

We can write the piecewise function as

$$p(Q) = \begin{cases} 60 - \frac{3}{2}Q & \text{for } Q \leq 20 \\ 50 - Q & \text{for } Q > 20 \end{cases}$$

Here is a graph illustrating this problem:



b. (3 pt.) Suppose that the marginal cost of mowing is 70 for each neighbor. What is the socially optimal total amount of mowing, Q_s? If Homer and Flanders play a static game, what are the Nash equilibrium quantities q_H^* and q_F^*?

Even at $Q = 0$, the marginal cost of mowing exceeds the social marginal benefit from mowing, which means that the socially optimal amount of mowing is $Q_s = 0$. If this is a static game, then both neighbors decide not to mow at all. To see this, notice that when $Q = 0$, each neighbor's private marginal benefit from starting to mow is below the marginal cost of mowing, so not mowing is a dominant strategy for each player. We end up with zero provision of the public good. (In this case, however, that's a good thing!)

c. (3 pt.) Now suppose the marginal cost is 45 for each neighbor. What is the socially optimal total amount of mowing, Q_s? From the standpoint of Pareto efficiency, does it matter who does the mowing (and if so, which neighbor should do the mowing)? If Homer and Flanders play a static game, what are the Nash equilibrium quantities q_H^* and q_F^*?

The social optimum occurs at the point where the social marginal benefit equals the (social) marginal cost. Given the demand curve we found in part a, when the marginal cost is 45, the SMB equals the SMC on the first part of the social demand curve:

$$p(Q) = 60 - \frac{3}{2}Q = 45 \implies Q_s = 10$$

[Note: if you look for a solution on the second part of the social marginal benefit curve—that is, assuming that $Q > 20$—you'll obtain $Q = 5$, which contradicts the assumption that $Q > 20$. So this is an invalid answer.]

Since the marginal cost is the same for both Homer and Flanders, from the standpoint of Pareto efficiency it does *not* matter who does the mowing.

To find the Nash equilibrium quantities, notice that choosing zero is still a dominant strategy for Flanders. Knowing that Flanders will set $q_F^* = 0$, Homer will choose q_H^* to satisfy $p_H(Q) = 50 - q_H = 45 \implies q_H^* = 5$. Now we have some provision of the public good (5 units total), but less than the optimal amount (10 units).

d. (3 pt.) Finally, suppose that Homer's marginal cost is 20, while Flanders's marginal cost is only 5. What is the socially optimal amount Q_s? From the standpoint of Pareto efficiency, does it matter who does the mowing (and if so, which neighbor should do the mowing)? What are the Nash equilibrium quantities q_H^* and q_F^*?

From the standpoint of Pareto efficiency, it is better to have Flanders do all of the mowing, since his marginal cost is lower. With Flanders doing all of the mowing, the socially optimal quantity occurs at the point where the social marginal benefit equals Flanders's marginal cost. This intersection occurs on the second part of the demand curve, where only Homer has a positive marginal benefit:

$$p(Q) = 50 - Q = 5 \implies Q_s = 45$$

Now let's think about the Nash equilibrium. This problem is tricky because neither player has a dominant strategy of "not mowing at all". Notice, however, that Flanders will certainly choose $q_F^* \leq 10$, since choices $q_F > 10$ are strictly dominated (in this range, Flanders's private marginal cost always exceeds his private marginal benefit). But if $q_F^* \leq 10$, then Homer's best response is to choose $q_H^* = 30 - q_F^*$, since

$$p_H(Q) = 50 - q_F^* - q_H^* = 20 \implies q_H^* = 30 - q_F^*$$

With $q_F^* \leq 10$, the expression $q_H^* = 30 - q_F^*$ equals at least 20. With Homer contributing at least 20 units of the public good, however, Flanders's private marginal benefit equals 0: so Flanders's best response to Homer is to choose $q_F^* = 0$. The Nash equilibrium strategies are: $q_H^* = 30$ and $q_F^* = 0$.

