Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Homework #7

Due: Saturday, December 7th at 5:00pm

1 Risk attitudes (6 pts. total)

For each of these utility functions, determine whether the agent is risk-averse, risk-neutral, or risk-loving (assume that wealth is positive: $w > 0$). Also indicate whether the agent would always buy, never buy, or be indifferent towards an actuarially fair insurance policy.

a. (2 pts.) $u(w) = 3w^4 + 2w^2$

$u'(w) = 12w^3 + 4w$ and $u''(w) = 36w^2 + 4$. Since $u''(w) > 0$, these are risk-loving preferences. The agent would not want to buy actuarially fair insurance.

b. (2 pts.) $u(w) = 3$

$u'(w) = 0$ and $u''(w) = 0$. Since $u''(w) = 0$, the agent is risk-neutral and is indifferent towards actuarially fair insurance.

c. (2 pts.) $u(w) = 4\sqrt{w} + \ln(w)$

$u'(w) = 2w^{-\frac{1}{2}} + \frac{1}{w}$ and $u''(w) = -w^{-\frac{3}{2}} - \frac{1}{w^2}$. Since $u''(w) < 0$, this agent is risk-averse. She would want to buy an actuarially fair insurance policy.

2 Wildfire risks (12 pts. total)

A California resident gets utility from wealth (w) and health (h):

$$u(w, h) = \sqrt{w} + \sqrt{h}$$

She starts out with wealth $w_0 > 0$ and health $h_0 = 100$, but there is a 25% chance that wildfires will create air pollution, lowering her health to $h = 36$. Before wildfire season starts, she can choose to install an HVAC system that would completely eliminate this risk.

a. (3 pts.) If the resident has initial wealth $w_0 = 144$, what is her expected utility without the HVAC system? How much would she be willing to pay for the HVAC system?

Without the HVAC, expected utility is

$$\mathbb{E}(u(w,h)) = \frac{1}{4} \times (\sqrt{144} + \sqrt{36}) + \frac{3}{4} \times (\sqrt{144} + \sqrt{100}) = \frac{18}{4} + \frac{66}{4} = 21$$

If the agent buys an HVAC system at price x, her utility is no longer random: it equals

$$u(w_0 - x, h_0) = \sqrt{144 - x} + \sqrt{100} = \sqrt{144 - x} + 10$$

The agent's WTP for HVAC is the price x^* that makes her indifferent between buying and not buying the HVAC:

$$\underbrace{21}_{\text{EU w/o HVAC}} = \underbrace{\sqrt{144 - x^*} + 10}_{\text{EU with HVAC}} \implies \sqrt{144 - x^*} = 11 \implies 144 - x^* = 121 \implies x^* = 23$$

b. (3 pts.) If the resident instead has initial wealth $w_0 = 256$, how much would she be willing to pay for HVAC? Comparing your answers to parts a and b, what does this suggest about richer/poorer people's ability to cope with environmental risks?

Now the calculation is

$$\underbrace{\frac{1}{4}(\sqrt{256} + \sqrt{36}) + \frac{3}{4}(\sqrt{256} + \sqrt{100})}_{\text{EU w/o HVAC}} = \underbrace{\sqrt{256 - x^*} + 10}_{\text{EU with HVAC}}$$

Solving this equation for x yields the new WTP $x^* = 31$.

So, our answers to parts a and b suggest that richer CA residents will have a higher WTP for the HVAC system (i.e., be in a better financial position to deal with environmental risks). Intuitively, the poorer resident has a bigger marginal utility from consumption of wealth, so she suffers more when she sacrifices consumption in an effort to protect her health from wildfire-induced air pollution.

c. (3 pts.) Suppose that—as the climate continues to change—the probability of wildfire-induced health damages rises to 50%. If the price of an HVAC system is 14, find the "cutoff" value w^* such that residents with $w_0 > w^*$ purchase the HVAC and residents with wealth $w_0 < w^*$ do not.

Let's find the wealth w^* for which the resident is indifferent at an HVAC price of 14:

$$\underbrace{\sqrt{w^*-14}+\sqrt{100}}_{\text{EU with HVAC}} = \underbrace{\frac{1}{2}(\sqrt{w^*}+\sqrt{100})+\frac{1}{2}(\sqrt{w^*}+\sqrt{36})}_{\text{EU w/o HVAC}}$$

$$\sqrt{w^*-14}+10 = \sqrt{w^*}+8$$

$$\sqrt{w^*-14} = \sqrt{w^*}-2$$

$$\text{(square both sides)} \qquad w^*-14 = w^*-4\sqrt{w^*}+4$$

$$4\sqrt{w^*} = 18$$

$$w^* = \frac{81}{4} = 20.25$$

So, when the price is 14, "rich" residents (those with $w_0 > 20.25$) will buy the HVAC; "poor" residents (those with $w_0 < 20.25$) will not. The problem illustrates a real-world feature of environmental crises like California's wildfires: poorer households are less likely to have the financial resources needed to minimize their exposure to air pollution through the use of expensive filters, respirator masks, moving to another city, or other ways of limiting their exposure.

d. (3 pts.) Now suppose health is non-random (equaling $h_0 = 100$), but wildfires create financial risk: there is a 20% probability that $w = 25$, otherwise $w = 225$. Compute the mean and variance of w. Then compute the resident's certainty equivalent.

The expected value of wealth equals

$$\mathbb{E}(w) = \frac{1}{5}\times 25 + \frac{4}{5}\times 225 = 185$$

The variance of wealth equals

$$\text{Var}(w) = \frac{1}{5}\times(25-185)^2 + \frac{4}{5}\times(225-185)^2 = \frac{1}{5}\times 25600 + \frac{4}{5}\times 1600 = 6400$$

The certainty equivalent is the value of wealth CE defined by the indifference equation

$$\sqrt{CE}+\sqrt{100} = \frac{1}{5}(\sqrt{25}+\sqrt{100})+\frac{4}{5}(\sqrt{225}+\sqrt{100})$$

(The health-related terms $\sqrt{100}$ cancel out: the level of health has no impact on the certainty equivalent in this problem.)

Simplifying and solving for the certainty equivalent yields $CE = 169$: the resident is indifferent between having wealth of 169 for sure vs. playing a "lottery" in which wealth is randomly equal to either 25 or 225.

3 The value of information (12 pts.)

When making decisions in the presence of uncertainty, agents may have an incentive to invest in information that would reduce this uncertainty and hence help them make better decisions. But information is only useful when there's a chance it will alter your decision.

Pfizer is deciding whether to develop a promising new hair-loss medication called Lycanthea. To develop Lycanthea, Pfizer must pay a fixed cost $FC \geq 0$. Once the drug is developed, manufacturing additional doses is nearly costless, so let's assume that $VC(Q) = 0$.

If Pfizer develops Lycanthea, it will be a uniform-pricing monopolist. However, there is uncertainty about consumer demand. Early-stage clinical trials suggest that there is a 25% chance Lycanthea has no side effects, in which case Pfizer would face high demand given by

$$p_h(Q) = 20 - Q$$

However, based on a troubling incident reported during the early trials, Pfizer's scientists estimate there is a 75% chance that Lycanthea increases the risk of spontaneously turning into a werewolf. That would generate negative press coverage, lowering demand to

$$p_l(Q) = 8 - Q$$

If Pfizer decides to pay the fixed cost, it immediately learns what demand curve it faces (i.e., before having to choose Q). Pfizer is risk-neutral: it wants to maximize its expected profit.

a. (2 pts.) Suppose that (despite the werewolf risk) Pfizer decides to enter the market. Compute Pfizer's optimal quantity Q_l^* in the event that demand is low ($p(Q) = 8 - Q$). Compute Pfizer's optimal quantity Q_h^* in the event demand is high ($p(Q) = 20 - Q$).

   This is a standard uniform-monopoly problem. If demand is low, Pfizer solves

   $$\max_Q (8-Q)Q - FC \implies \text{FOC}: 8-2Q = 0 \implies Q_l^* = 4 \implies p_l^* = 4 \implies \pi_l = 16 - FC$$

   (where I've calculated π since we'll need this shortly). If demand is high, Pfizer solves

   $$\max_Q (20-Q)Q - FC \implies \text{FOC}: 20-2Q = 0 \implies Q_h^* = 10 \implies p_h^* = 10 \implies \pi_h = 100 - FC$$

   (Once Pfizer decides to enter, the fixed cost is sunk, so it doesn't affect Q_l^* or Q_h^*.)

b. (2 pts.) If $FC = 0$, Pfizer enters the market. Compute its expected profit.

   Pfizer's expected profit is just a weighted average of its profits in each of the two cases (high demand or low demand):

   $$\mathbb{E}(\pi) = \frac{1}{4} \times \pi_h + \frac{3}{4} \times \pi_l = \frac{1}{4} \times 100 + \frac{3}{4} \times 16 = 25 + 12 = 37$$

c. (3 pts.) If $FC = 10$, will Pfizer enter the market? If $FC = 20$? If $FC = 40$?

Building on our previous calculation, Pfizer's expected profits from entering are $\mathbb{E}(\pi) = 37 - FC$. Thus Pfizer will enter if $FC = 10$ or if $FC = 20$, but not if $FC = 40$.

Now suppose that, before deciding whether to pay the fixed cost FC, Pfizer can run another clinical trial to determine (with perfect accuracy) whether Lycanthea causes werewolves.

d. (4 pts.) For each of the following, determine how much (i.e., the maximum amount that) Pfizer would be willing to pay for another clinical trial.

i. $FC = 10$

WTP = 0. In this case, Pfizer develops Lycanthea regardless of whether Lycanthea creates werewolves (since even in the event of "bad news" Lycanthea's profits—ignoring the sunk cost of the clinical trial—will equal $\pi_l = 16 - 10 = 6 > 0$), so paying for the extra trial doesn't improve Pfizer's decision-making: it's just a waste of money.

ii. $FC = 20$

WTP = 3. If Pfizer doesn't conduct the trial, then as we established in part c, Pfizer will develop Lycanthea and make expected profit $\mathbb{E}(\pi) = 37 - FC = 17$.

If Pfizer conducts the trial, however, it will only develop Lycanthea if it gets "good news". With 25% probability, Pfizer learns that demand will be high. In this case, Pfizer enters the market and makes profit $\pi_h = 100 - 20 - CT = 80 - CT$, where CT is the cost of the clinical trial. With 75% probability, Pfizer learns that demand will be low; in this case, it stays out of the market and makes profit $\pi_l = 0 - CT = -CT$.

Pfizer's WTP for the trial is the value of CT that makes Pfizer indifferent between these two courses of action:

$$\underbrace{17}_{\text{expected profits w/o trial}} = \underbrace{\frac{1}{4}(80 - CT) + \frac{3}{4}(-CT)}_{\text{expected profits with trial}} = 20 - CT \implies CT^* = 3$$

iii. $FC = 40$

WTP = 15. In this case, if Pfizer doesn't conduct the trial, then as we established in part c, Pfizer <u>won't</u> develop Lycanthea and its profit will be zero.

If Pfizer conducts the trial, then—as we calculated in part d(ii)—Pfizer will develop Lycanthea if and only if gets "good news". So, if Pfizer conducts the extra trial, its expected profit equals

$$\mathbb{E}(\pi) = \frac{1}{4}\underbrace{(100 - FC - CT)}_{\text{good news}} + \frac{3}{4}\underbrace{(0 - CT)}_{\text{bad news}} = 15 - CT$$

So, Pfizer's WTP for the trial is the value of CT that makes Pfizer indifferent between these two courses of action:

$$\underbrace{0}_{\text{expected profits w/o trial}} = \underbrace{15 - CT}_{\text{expected profits with trial}} \implies CT^* = 15$$

In effect, the clinical trial is a low-cost way for Pfizer to find out whether there's a good market opportunity here—usually the answer is no, but Pfizer can make a lot of profit if the answer is yes, so the information is very valuable.

iv. $FC = 120$

WTP = 0. In this case, Pfizer never develops Lycanthea (even if it doesn't cause werewolves), so (as with $FC = 10$) the extra trial wouldn't influence Pfizer's decision—it would just be a waste of money.

(Hint: in each case, determine what Pfizer will do with/without the information.)

e. (1 pt.) Now suppose Pfizer can't conduct another clinical trial, but it can perfectly price discriminate if it enters the market. If $FC = 40$, will Pfizer enter the market?

Since the marginal cost of production is zero, and since Pfizer can engage in perfect price discrimination, if it enters the market Pfizer will sell to everyone with positive willingness to pay. If Pfizer enters and demand is high ($p(Q) = 20 - Q$), Pfizer's profits will be 200 (the revenue) minus 40 (the fixed cost) = 160. If Pfizer enters and demand is low ($p(Q) = 8 - Q$), Pfizer's profits will be 32 (the revenue) minus 40 (the fixed cost) = −8. So, if Pfizer enters the market, its expected profit is

$$\mathbb{E}(\pi) = \frac{1}{4} \times 160 + \frac{3}{4} \times (-8) = 40 - 6 = 34$$

Since entering yields positive expected profit, Pfizer will want to enter the market.

