<u>Lecture Note 3: Monopoly Pricing</u>

monopoly: the only supplier of a good that has no close substitute

- "only supplier": other firms are unable or unwilling to produce it
- "no close substitute": consumers have few alternatives

Real-world examples:

- Prescription drugs (until their patents expire)
- Innovative products (until they get copied)
- Local utilities (electricity, internet)
- Social media platforms

Why are certain markets monopolized?

- One firm may have a big **technological advantage** over other firms.
  - Example: Ford pioneered the use of assembly lines to make cars.
- Economies of scale may lead to a **natural monopoly**.
  - Example: it's not profitable for two utilities to build electric grids.
- **Network externalities** can make the first entrant very powerful.
  - Example: the more people using Venmo, the better Venmo gets.
  - Very relevant today: think about Facebook, Twitter, Airbnb.
- Government policies may create **barriers to entry**.
  - Patent: the exclusive right to sell a product for a limited time.
  - License: the (non-exclusive) right to participate in a market.

In this lecture, we'll try to understand how monopolists price their products. Next time, we'll explore how monopolies affect the size of the economic pie—and what we can do about them.

Maximizing profits under monopoly

For a perfectly competitive firm, the profit maximization problem is

$$\max_q \ \pi(q) = pq - C(q)$$

Notice that p is a constant here. Why? Because the firm is a price-taker.

For a monopolist, the profit maximization problem looks like this:

$$\max_Q \ \pi(Q) = \underbrace{p(Q)Q}_{\text{total revenue}} - \underbrace{C(Q)}_{\text{total cost}}$$

Since the amount supplied by the monopolist (q) equals the amount supplied in the market overall (Q), I've written Q instead of q here.

Key difference: price is a function of quantity!

- Since a monopolist is the only firm in the market, it isn't "small".
- To sell more units, the monopolist has to lower its price: $p'(Q) < 0$. A monopolist faces a **downward-sloping demand curve**.
- We're assuming here that the monopolist charges the same price, $p(Q)$, for each of the Q units sold. This is important, and we'll revisit it soon.

Let's start by taking the first-order condition (FOC): using the product rule,

$$\frac{d\pi}{dQ} = p(Q) + p'(Q)Q - C'(Q) = 0$$
$$\implies \underbrace{p(Q^*) + p'(Q^*)Q^*}_{\text{marginal revenue}} = \underbrace{C'(Q^*)}_{\text{marginal cost}}$$

Technically, we also have to check the second-order condition (SOC):

$$\frac{d^2\pi}{dQ^2} = \frac{dMR}{dQ} - \frac{dMC}{dQ} \quad \text{needs to be negative}$$

But the SOC will hold in the problems we study, and we won't focus on it.

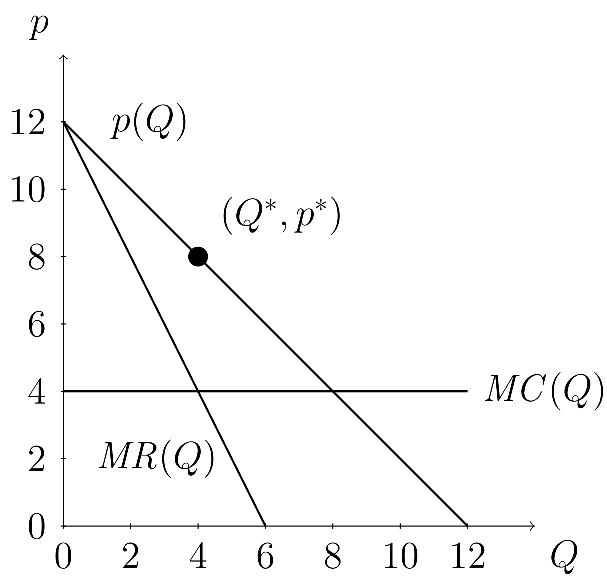
A numerical example

Example: Suppose the monopolist faces the demand curve $p(Q) = 12 - Q$ and has the linear cost function $C(Q) = 4Q$.

What are the monopoly's optimal quantity Q^* and price p^*?

- Choose Q to maximize profits: $\max_{Q} \ \pi(Q) = (12 - Q)Q - 4Q$
- Take the FOC: $\frac{d\pi}{dQ} = 12 - Q^* - Q^* - 4 = 0$
- Solve for quantity: $2Q^* = 8 \implies Q^* = 4$
- Then solve for price: $p^* = 12 - Q^* = 8$

Here's the graphical representation:



Since any choice of Q automatically implies a choice of p, you might ask: why can't the monopolist just choose p from the start? Well, she can:

- Choose p to maximize profits: $\max_{p} \ \pi(p) = p(12 - p) - 4(12 - p)$
- Take the FOC: $\frac{d\pi}{dp} = 12 - p^* - p^* - 4(-1) = 0$
- Solve for price: $2p^* = 16 \implies p^* = 8$ as before
- Then solve for quantity: $Q^* = 4$ as before

The marginal revenue from selling one more unit

Now, we've seen that the monopolist's marginal revenue is

$$MR(Q) = p(Q) + p'(Q)Q$$

That's what the math tells us. What's the economics?

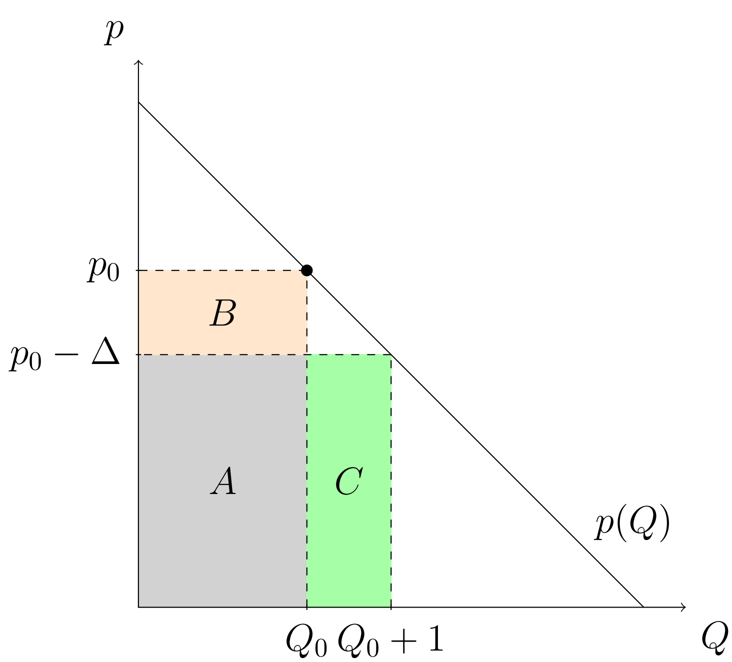
I am a monopolist in the production of Professor Price's ECN 100B lectures. Suppose I start selling tickets to attend class (don't worry, I'm not allowed).

Imagine I'm planning to sell Q_0 tickets at a price p_0.
What happens if I decide to sell one more ticket?

- Good news: I get approximately p_0 from selling this "marginal" ticket.
- Bad news: I have to lower the price for the Q_0 "inframarginal" units.

Why can't I give a discount only on the marginal ticket? Remember: right now, we're assuming that I have to charge everyone the same price.

It's easiest to see this graphically:



- Initially: sell Q_0 at price p_0
- $\implies$ old revenue $= A + B$
- Now sell $Q_0 + 1$ at price $p_0 - \Delta$
- $\implies$ new revenue $= A + C$
- Marginal revenue $= C - B$
- MR may be positive or negative

The (price) elasticity of demand

To understand what price a monopoly will charge, it's helpful to think about the **price elasticity of demand** (or "elasticity of demand" for short)—the % change in quantity demanded in response to a 1% increase in price.

If x changes from x to $x + dx$, the percentage change is

$$\frac{(x + dx) - (x)}{x} = \frac{dx}{x}$$

The percentage change in Q is $\frac{dQ}{Q}$. The percentage change in p is $\frac{dp}{p}$.

So the elasticity of demand is $\frac{dQ}{Q} / \frac{dp}{p} = \frac{dQ}{dp}\frac{p}{Q}$.

We write this as $\varepsilon = \frac{dQ}{dp}\frac{p}{Q}$. Even though we usually just write "ε", don't forget that the elasticity of demand is typically a function of Q—the elasticity changes depending on where we are on the demand curve.

Example: Suppose that $p(Q) = 12 - Q$. Compute ε as a function of Q.

- Start by finding $Q(p)$: $Q(p) = 12 - p$
- Then compute $\frac{dQ}{dp}$: $\frac{d}{dp}(12 - p) = -1$
- Put it all together: $\varepsilon = \frac{dQ}{dp}\frac{p}{Q} = (-1)\frac{12-Q}{Q} = 1 - \frac{12}{Q}$

What does ε mean?

- Is $\varepsilon \lesseqgtr 0$? Since demand slopes downward, $\frac{dQ}{dp} \leq 0$ and thus $\varepsilon \leq 0$.
- When $|\varepsilon|$ is large, consumers are more price-sensitive.

You should be familiar with three special cases:

- $\varepsilon = 0$: demand is **perfectly inelastic**
- $\varepsilon = -\infty$: demand is **perfectly elastic**
- $\varepsilon = -1$: demand is **unit elastic**

Exercise: In the problem above, at what quantity Q is demand perfectly inelastic? perfectly elastic? unit elastic? (Answers: 12, 0, 6, respectively.)

Price markups and the elasticity of demand

We can rearrange the monopoly FOC to obtain the following equation:

$$\frac{p - MC}{MC} = -\frac{1}{1+\varepsilon}$$

(Deriving this equation takes several steps, and the details are not very important for our purposes. I've included the derivation in an appendix at the end of this lecture note, but you won't be tested on it.)

The term on the lefthand side of this equation is called the **price markup**: it tells us how much a good is "overpriced" relative to the cost of production. If the markup is 10%, then the price is 10% bigger than the marginal cost.

The term on the righthand side of this equation is

- small when consumers are very price-sensitive (ε is close to $-\infty$)
- large when consumers aren't very price-sensitive (ε is close to 0)

Intuitively, monopolists "overcharge" their customers when they don't lose many customers from doing so. When consumer demand is more elastic, monopolists set prices closer to marginal cost.

The markup equation is quite useful. If we know a monopolist's price (p) and marginal cost (MC), we can rearrange the formula to figure out what elasticity (ε) it must be facing. If we know the elasticity of demand, we can predict what price markup the monopolist will choose.

Exercise: If a monopolist sets its price 10% above its marginal cost, what elasticity of demand does it face? What if the price markup is 100%?

We can solve the markup equation for ε to obtain: $\varepsilon = -\frac{1}{\text{markup}} - 1$. If the markup is 10%, then $\varepsilon = -\frac{1}{.1} - 1 = -11$. If the markup is 100%, then $\varepsilon = -\frac{1}{1} - 1 = -2$. When the markup is bigger, demand must be less elastic.

Monopolists set prices where demand is relatively elastic

Here's the markup equation again:

$$\frac{p - MC}{MC} = -\frac{1}{1+\varepsilon}$$

If $\varepsilon < -1$, we say that demand is **relatively elastic**. In this case,

$$-\frac{1}{1+\varepsilon} > 0 \implies \frac{p - MC}{MC} > 0 \implies p > MC$$

which makes sense.

But what if $\varepsilon \geq -1$?

- $\varepsilon = -1 \implies$ markup $= -\frac{1}{0}$ (undefined)
- $\varepsilon = -\frac{1}{2} \implies$ markup $= -200\%$
- $\varepsilon = 0 \implies$ markup $= -100\%$

But a monopolist would never set $p < MC$. What's going on here?

Thought experiment: Suppose demand for EpiPens were perfectly inelastic at every possible price. Right now Mylan charges \$600 for a pack of EpiPens. What should do it do? Keep raising p and make infinitely large profits!

Now suppose demand for EpiPens is **relatively inelastic** ($-1 < \varepsilon < 0$). What if Mylan raises its price by 1%?

- By definition, quantity demanded falls by ε%, which is less than 1%.
- Since price rises more than quantity falls, total revenue must go up.
- Furthermore, since quantity falls, total production costs go down.
- Since revenue goes up and costs go down, profits must go up.

So, if a monopolist starts at a "candidate price" where demand is relatively inelastic, it can "profitably deviate" by increasing its price. The optimal price must be at a point on the demand curve where $\varepsilon < -1$.

Fixed costs, variable costs, and backward induction

fixed cost: a cost that does not vary with the level of output
variable cost: a cost that does vary with the level of output

Example: Suppose I'm deciding whether to open an ice cream shop.

- Fixed costs: cost of renting the building, cost of machines
- Variable costs: cost of ingredients

After doing some market research, I determine that:

- I'd face the (inverse) demand curve $p(Q) = 12 - Q$.
- I would have to pay a fixed cost $FC = 20$ up front.
- I would have variable costs $VC(Q) = \frac{1}{2}Q^2 + 3Q$.

I have two (related) decisions to make:

1. Should I enter the market?
2. If I enter the market, how much ice cream should I produce?

Seems like the entry decision comes first. Right?

Wrong! It's a natural way to approach the problem, but not the best way.

- Economists like to work backwards.
- I need to answer the "second" question to answer the "first" one.
- I need to know what price I'd set to know if I can make positive profit.

This argument is an example of **backward induction**. We'll see this a lot more when we get to game theory.

When to enter a market

Step 1: Decide how much to produce *if I choose to produce at all.*

- Standard monopoly problem:

$$\max_{Q} \; \pi(Q) = (12 - Q)Q - \frac{1}{2}Q^2 - 3Q - 20$$

- FOC: $12 - 2Q - Q - 3 = 0 \implies Q^* = 3 \implies p^* = 9$
- Key observation: Q^* doesn't depend on fixed costs! Since the fixed cost is a constant, it goes away when we differentiate π. If I decide to produce, fixed costs don't affect *how much* I produce.

Step 2: Compute my profits if I produce.

- Compute profits using Q^* from Step 1.
- This is the most money I can possibly make if I produce.
- $\pi = 9 \cdot 3 - \frac{1}{2} \cdot 3^2 - 3(3) - 20 = \underbrace{27}_{TR} - \underbrace{13.5}_{VC} - \underbrace{20}_{FC} = -6.5$

Step 3: Compare to my profits if I don't enter the market.

- No revenues from selling ice cream, no costs from producing it.
- Perhaps there's some other business activity I'd want to pursue.
- But for today: let's assume I make zero profit if I don't enter.

What should I do? Since entering yields negative profit, and staying out yields zero profit, I should not enter this market.

When to call it quits

Suppose that, against my better judgment, I open an ice cream store anyway. Unfortunately, my market research was right: my store flops, I get a bunch of mean Yelp reviews, and I end up with negative profits, $\pi = -6.5$.

What should I do now?

It depends on whether my fixed costs are **sunk** or **recoverable**.

- If they're recoverable, I can get my money back by exiting the market.
  - Find a new tenant to take over my lease.
  - Sell my ice-cream machines at the original purchase price.
- If they're sunk, I can't get my money back.
  - Nobody wants my lease, so I have to keep paying rent.
  - Nobody wants to buy my crummy used machines.
- Costs may also be partially recoverable.
  - It takes a couple of months to find a replacement tenant.
  - I can sell my machines, but only at a steeply discounted price.

In this case:

- If my costs are recoverable: I should exit the market and get $\pi = 0$.
- If my costs are sunk: exiting the market would yield $\pi = -20$. It's better to stay in the market and cut my losses, making $\pi = -6.5$.

Exercise: Suppose that I can only recover a fraction x of my fixed costs. For what value of x am I indifferent between staying and exiting?

If I can recover a share x of the fixed costs, then the remaining share $1 - x$ is sunk, and my profits from exiting are $\pi = (-20) \cdot (1 - x)$. Since staying yields $\pi = -6.5$, I'm indifferent if both options give me the same profit:

$$-20 \cdot (1 - x) = -6.5 \implies x^* = 0.675 \text{ or } 67.5\%$$

Appendix: derivation of the monopoly price markup

To derive our equation for the monopoly's price markup, we start with the monopoly FOC:

$$p(Q) + p'(Q)Q = MC(Q)$$

Let's write $\frac{dp}{dQ}$ instead of $p'(Q)$, and let's simplify the notation by writing "$p(Q)$" instead of "p" and "MC" instead of "$MC(Q)$"—though we should bear in mind that the price and marginal cost still depend on Q.

$$p + \frac{dp}{dQ}Q = MC$$

Now let's multiply the second term in this equation by $\frac{p}{p}$:

$$p + p\frac{dp}{dQ}\frac{Q}{p} = MC$$

And now let's factor out p on the left side of the equation:

$$p\left(1 + \frac{dp}{dQ}\frac{Q}{p}\right) = MC$$

The term $\frac{dp}{dQ}\frac{Q}{p}$ looks a lot like an elasticity. In fact, it's the *inverse* price elasticity of demand, which is the reciprocal of the price elasticity:

$$\frac{dp}{dQ}\frac{Q}{p} = \frac{1}{\frac{dQ}{dp}\frac{p}{Q}} = \frac{1}{\varepsilon}$$

Plugging this expression into our equation above gives us:

$$p\left(1 + \frac{1}{\varepsilon}\right) = MC$$

With a little more rearranging, we can express the monopoly's markup as a function of the price elasticity of demand:

$$\frac{p - MC}{MC} = -\frac{1}{1 + \varepsilon}$$