Lecture Note 6: Factor Markets

Up to now: how much output to sell in the **product market**.
Today: how many inputs to purchase in the **factor market**.

Firms produce output using **factors of production**:

- Hiring labor.
- Renting or buying capital.
- Purchasing raw materials.

Product and factor markets are closely linked: changes that occur in one of these markets will often affect the other one.

- Price of soybeans $\uparrow \implies$ plant more seeds, hire more laborers.
- Boeing wins a military contract $\implies$ build labs, hire engineers.
- More college graduates $\implies$ shift towards high-tech products.

Lots of big contemporary debates are about factor markets:

- Will robots take our jobs?
- Would a $15 federal minimum wage create jobs, or destroy them?

These are the questions people ask labor economists at parties!

Producing output with a single factor

It's PSL season: Peet's is hiring baristas to make Pumpkin Spice Lattes.

- If it hires L workers, it can make $q(L)$ lattes, yielding revenue $R(q)$.
- $q(L)$ is the firm's **production function**: it tells us how much physical output a firm gets from a given choice of inputs.

Suppose that the Peet's profit-maximization problem is

$$\max_{L} \ \pi(L) = R(q(L)) - wL$$

where w is the wage. We're making two big assumptions here:

- We're assuming Peet's only hires labor. We can think of this as a "short-run" decision: for the moment, its capital stock is fixed.
- By writing wL, not $w(L)L$, we're assuming Peet's is a **wage-taker**: it can hire as many workers as it wants at a fixed wage. That is, it hires workers in a **competitive labor market**.*

Let's take the FOC. Using the chain rule,

$$\frac{d\pi}{dL} = \frac{dR}{dq}\frac{dq}{dL} - w = 0 \implies \frac{dR}{dq}\frac{dq}{dL} = w$$

We have names for these terms:

- $\frac{dq}{dL}$ is the **marginal physical product of labor (MPPL)**.

  (If we hire more labor, how many extra lattes can we make?)
- $\frac{dR}{dq}\frac{dq}{dL}$ is the **marginal revenue product of labor (MRPL).**

  (If we hire more labor, how much extra revenue do we get?)

Since $\frac{dR}{dq}$ is the marginal revenue, we can also write

$$MRPL = MR \times MPPL$$

*If an employer has to raise wages to attract more workers (that is, if $w'(L) > 0$), we say that it has *monopsony power*. Monopsony is a fascinating topic, but we won't cover it in this course.

Solving for L^* and doing comparative statics

Suppose the Peet's production function is

$$q(L) = A\sqrt{L}$$

where A is its "productivity" or "technology" (a positive constant).

Peet's sells lattes in a competitive product market at price $p > 0$, and it hires workers in a competitive labor market at wage $w > 0$.

Suppose we want to know how the optimal choice of labor will respond to an increase in (i) the price of a latte, (ii) productivity, or (iii) the wage.

- Step 1: write profits as a function of L.

$$\max_L \ \pi(L) = pA\sqrt{L} - wL$$

- Step 2: solve for L^* as a function of the parameters.

$$\text{FOC}: \quad \frac{pA}{2\sqrt{L}} - w = 0 \implies L^* = \left(\frac{pA}{2w}\right)^2$$

- Step 3: examine how changes in parameters affect L^*.
  - $p \uparrow \implies L^* \uparrow$ (since $\frac{dL^*}{dp} = \frac{pA^2}{2w^2} > 0$)
  - $A \uparrow \implies L^* \uparrow$ (since $\frac{dL^*}{dA} = \frac{p^2A}{2w^2} > 0$)
  - $w \uparrow \implies L^* \downarrow$ (since $\frac{dL^*}{dw} = -\frac{p^2A^2}{2w^3} < 0$)

If we plug in specific values of the parameters—for example, $p = 4$, $A = 10$, and $w = 10$—then we can figure out how many workers Peet's should hire.

But if we're interested in understanding what factors influence its choice, it's easiest to leave the parameters unspecified (i.e., as "p", "A", and "w") and calculate L^* as a function of the parameters.

Producing output with two factors

Now suppose that Peet's produces with a mix of labor and capital: $q(L, K)$.[†]

- We sometimes call this a "long-run" production function because both factors are free to adjust: Peet's can change both L and K.

Three special production functions are often used in economics:

1. **Linear production function.**
    - $q(L, K) = AL + BK$, where A and B are constants.
    - Firms can produce using L only, K only, or a mix of both.
    - e.g., CVS can use either human cashiers (L) or machines (K) for checkout. If $A > B$, humans are more productive.
    - (L, K) are *perfect substitutes*: we can swap one for the other.
2. **Leontief production function.**
    - $q(L, K) = \min\{AL, BK\}$, where A and B are constants.
    - Firms need B workers for every A machines, so that $AL = BK$. Otherwise, one of the factors is being (partially) wasted.
    - e.g., a taxi service needs one driver per cab ($A = B$).
    - (L, K) are *perfect complements*: we have to use them in a fixed proportion that depends only on A and B.
3. **Cobb-Douglas production function.**
    - $q(L, K) = AL^{\alpha}K^{1-\alpha}$. For $\alpha = \frac{1}{2}$, this is $q(L, K) = A\sqrt{LK}$.
    - Both L and K are needed to produce output, but the precise capital-labor ratio a firm uses is somewhat flexible.
    - (L, K) are *neither* perfect substitutes nor perfect complements.

[†] In some cases, it's better to think of the two factors of production as different types of workers: medical care is produced using doctors and nurses; grading is done by professors and TAs.

Maximizing profits in two steps

With two factors, firms use a two-step process to maximize profits:

- Step 1: **Cost minimization**: Find the cheapest combination of factors to produce any given level of output q.
- Step 2: **Optimal output**: Find the most profitable choice of q, given that output is produced as cheaply as possible.

We'll focus mostly on the cost-minimization step.

The cost-minimization problem

In general, the cost minimization problem is written as:

$$\min_{L,K} \; wL + rK \quad \text{subject to the constraint} \quad q(L,K) = \overline{q}$$

where w is the wage rate and r is the rental rate of capital (both constant).

The best way to solve this problem depends on the production function.

- Linear: we usually end up at a corner solution (only L or only K), unless the factors happen to be equally cost-effective. Since we won't be at an interior solution, FOCs won't be helpful here. Instead, we'll figure out which factor offers the most "bang for your buck".
- Leontief: the "min" function ($\min\{AL, BK\}$, i.e. the smaller of AL and BK) isn't differentiable when $AL = BK$, so calculus won't help here either. Instead, we'll use a "zero waste" argument.
- Cobb-Douglas: here, finally, we can use calculus. Since we're solving a *constrained* optimization problem, we'll solve by substituting the constraint ($q(L,K) = \overline{q}$) into the objective function ($wL + rK$).

Minimizing costs with each production function

Suppose that $w = 20$, $r = 10$, and we want to produce 1 unit output. For each production function, what's the cheapest way to do this? What happens if the price of capital falls from $r = 10$ to $r = 5$ (or even lower)?

1. Linear: suppose $q(L, K) = 4L + K$.
   - How to solve: which factor gives the most output per \$ spent?
   - If $r = 10$, labor is 4x as productive, but only 2x as expensive, so it's cheaper to produce using labor only: $L^* = \frac{1}{4}$, $K^* = 0$.
   - If r falls below 5, however, the firm switches to $L^* = 0$, $K^* = 1$.

2. Leontief: suppose $q(L, K) = \min\{4L, K\}$.
   - How to solve: set the two "min" terms equal: $AL = BK$.
   - Optimal choice: $L^* = \frac{1}{4}$, $K^* = 1$. (Anything extra is wasteful.)
   - The capital-labor ratio is fixed: if r falls, $\frac{K^*}{L^*}$ remains unchanged.

3. Cobb-Douglas: suppose $q(L, K) = \sqrt{LK}$.
   - How to solve: the firm solves the cost minimization problem

$$\min_{L,K} \; wL + rK \quad \text{subject to } \sqrt{LK} = 1$$

   - The constraint implies that $K = \frac{1}{L}$. Substituting this in:

$$\min_{L} \; wL + r\frac{1}{L}$$

   - Now we can take an FOC:

$$w - r\frac{1}{L^2} = 0 \implies L^* = \sqrt{r/w}, \; K^* = \sqrt{w/r}, \; \frac{K^*}{L^*} = \frac{w}{r} = 2$$

   - If r falls from 10 to 5, then $\frac{K^*}{L^*}$ rises from 2 to 4.

Scale and substitution effects: or, will robots take our jobs?

Technology is rapidly advancing. As technologies like machine learning and 3D printing mature in the coming years, will robots take our jobs?

We can think of these technologies as reducing the effective cost of capital. What will happen to overall US employment as the cost of capital declines?

Answer: it depends! There are two effects that go in opposite directions:

- Substitution effect: as the price of capital (r) falls relative to the price of labor (w), firms substitute away from labor and towards capital, so the capital-labor ratio K^*/L^* goes up. The strength of this effect depends on how easily firms can substitute between factors:
  - Linear production function: very strong substitution effect.
  - Leontief production function: no substitution effect at all.
  - Cobb-Douglas: moderately strong substitution effect.
- Scale effect: as the price of capital falls, the overall cost of production goes down. As a result, firms expand their output (increasing the "scale" of operations), which requires more of both capital and labor. For example, a business might open a second location.

The net effect is **theoretically ambiguous** (it could go either way). Which effect dominates is an **empirical question** (we need data).

Our theories and models help us think rigorously about the world ...
... but we have to test them against real-world data.

Many economists (including me) spend their days analyzing data.

This class is very theoretical, but empirical work plays a bigger role in many of UC Davis's upper-division economics electives. Take them!

