Lecture Note 7: Static Games with Pure Strategies

Game theory is the study of *strategic interactions*, in which each "player" has to think about what each of the other players is going to do.

- My optimal strategy depends on what I expect you to do.
- Your optimal strategy depends on what you expect me to do.

Sometimes strategic interactions are not too important.

- Price-takers don't care what each individual market participant does: they just care about the overall market price.
- Monopolists like PG&E don't have any competitors to worry about. They care about total consumer demand, but not specific consumers.

But strategic interactions are common in sports/economics/politics/life.

- Soccer players deciding whether to kick the ball left or right.
- Coke and Pepsi deciding how much to advertise.
- US and North Korea deciding whether to conduct military exercises.
- Roommates deciding whether to be neat or messy.

We can describe each of these situations as a **game**. Early research into game theory focused on card games and other games of chance, which is where the name "game theory" comes from. But as the US–North Korea example shows, some "games" are quite serious for the players involved.

The goal of game theory is to *predict the outcomes* of games like these: given the "rules" of the game, what will each player do?

This lecture note and the next one will cover **static games**, in which the players move at the same time. After that, we'll cover **dynamic games**, in which the players take turns moving.

Example 1: Prisoner's Dilemma

Prisoner's Dilemma is the most famous game of them all.

- Bonnie and Clyde enjoy robbing banks. This time they get caught!
- The police have enough evidence to convict them both of weapons possession (a minor charge). But if one robber "flips" on the other, the police can convict the other of armed robbery (a major charge).
- The police put Bonnie and Clyde in separate cells and interrogate them. Each robber has two options: "Flip" or "Deny":
  - If both Deny: both go to prison for weapons possession.
  - If both Flip: both go to prison for armed robbery, with slightly reduced sentences for helping the police.
  - If one Flips and one Denies: the flipper goes free, and the flipped-upon unfortunately goes to prison for a long time.

We can represent this game using the **payoff matrix** below:

		Clyde	
		Flip	Deny
Bonnie	Flip	$0, 0$	$10, -2$
	Deny	$-2, 10$	$9, 9$

- Player 1's name is on the left (Bonnie), Player 2's is on top (Clyde).
- Each cell indicates player 1's payoff, followed by player 2's payoff. The exact numbers aren't too important; what matters is their order.
- Each player prefers a bigger payoff (or a less-negative one).
- We write their chosen strategies as an ordered pair. For example, if Bonnie flips and Clyde denies, we write "(Flip, Deny)".

Game theory terminology

A **game** is a description of a scenario in which two or more people interact. We call these people **players**. In ECN 100B, we'll always have two players.

Each player's choice set includes one or more actions, called **strategies**.

- If you commit to playing a single action, we call it a **pure strategy**.
- If you pick at random (e.g., flipping a coin), it's a **mixed strategy**.

At the end of the game, each player gets a **payoff**:

- "Payoff" just means utility (for people) or profit (for firms).
  - Bonnie's payoff is her utility. Coke's payoff is its profit.
- A player's payoff depends on the strategies chosen by <u>all</u> players.
  - Bonnie's payoff depends both on her decision and on Clyde's. Coke's payoff depends on its own advertising and on Pepsi's.
- Each player just cares about (tries to maximize) their own payoff.
  - Bonnie doesn't *directly* care about what payoff Clyde receives. She is neither altruistic towards him nor spiteful towards him.
  - But she cares about his potential payoffs *indirectly*, because they'll affect his behavior, which will affect her own payoff.

We assume that players have **common knowledge of rationality**:

- Every player knows the rules of the game (strategies, payoffs, etc.).
- Every player knows that every player knows the rules of the game.
- Every player knows that every player knows that every player knows the rules of the game ... and so on, and so on, and so on ...

With common knowledge, it's as though each player has taken game theory and assumes the other player has taken game theory too.

Static games

In a **static game**,

- Each player moves only once.
- Both players move at the same time.
- Players pick their strategies without knowing what the other will do.
- However: each player can try to predict what the other one will do.

This is a good description of the story behind Prisoner's Dilemma.

Example 1 (continued): Prisoner's Dilemma

Here's Prisoner's Dilemma again:

		Clyde: Flip	Clyde: Deny
Bonnie	Flip	0, 0	10, -2
	Deny	-2, 10	9, 9

Which strategy will Bonnie choose? Which strategy will Clyde choose?

- Suppose Bonnie thinks Clyde will Flip (so we're in the left column). If Bonnie Flips too, she gets 0. If she Denies, she gets -2.
  $\Longrightarrow$ Bonnie should Flip.
- Suppose Bonnie thinks Clyde will Deny (we're in the right column). If Bonnie Flips, she gets 10. If she Denies, she gets 9.
  $\Longrightarrow$ Bonnie should Flip.
- Clyde comes to the same conclusion: he should Flip no matter what!
- Bonnie and Clyde end up getting the payoffs $(0, 0)$. Not so good!

It's a tragic end for our poor robbers: by each pursuing their self-interest, they both end up worse off than they'd be if they'd both chosen Deny!

<u>Strictly dominant strategies</u>

A strategy is **strictly dominant** if it yields a higher payoff than any other available strategy, no matter what the other player does.

- In Prisoner's Dilemma, Flip is strictly dominant for both players.
- If a player has a strictly dominant strategy, she always chooses it.
- No matter what I expect the other player to do, playing a strictly dominant strategy is a good idea: *holding the other player's action fixed*, the dominant strategy gives me the best possible payoff.

A strategy is **strictly dominated** if there's some alternative strategy that always yields a higher payoff, no matter what the other player does.

- In Prisoner's Dilemma, Deny is strictly dominated for both players.
- If a player has a strictly dominant strategy, she *never* chooses it.[1]
- No matter what I expect the other player to do, playing a strictly dominated strategy is a bad idea.

Strictly domin<u>ant</u> strategies are good. Strictly domin<u>ated</u> ones are bad.

A player can have a strictly dominated strategy without having a strictly dominant one. In soccer, sometimes kicking left is optimal, sometimes kicking right is optimal, but we can all agree that kicking the ball into your own net is strictly dominated by kicking it either left or right.

If both players have a strictly dominant strategy, we can predict exactly what will happen: we call the outcome a **dominant strategy solution**.

- In Prisoner's Dilemma, (Flip, Flip) is a dominant strategy solution.

In most games, however, nobody has a dominant strategy. Games where *everyone* has a dominant strategy are even rarer. If there is no dominant strategy solution, we need some other way to predict what will happen.

[1] I've fixed a typo here: the original version of the notes incorrectly said "dominated" here.

Example 2: bike accident!

Every day at UC Davis, game theory professors (on foot) and game theory students (on bikes) play the following dangerous game:

		Bicyclist	
		Focus	Text
Professor	Sidewalk	$1, 2$	$1, 5$
	Road	$1, 2$	$-5, 3$

Let's look for a dominant strategy solution ...

- Does Professor have a strictly dominant strategy? No.
- Does Bicyclist have a strictly dominant strategy? Yes, "Text".

"Sidewalk" isn't a strictly dominant strategy because "Road" works fine if Bicyclist plays Focus. However, "Sidewalk" is clearly better if Bicyclist plays Text. We therefore say that "Sidewalk" is weakly dominant.

- A **weakly dominant strategy** never performs worse than any other strategy, and there is at least one case where it performs better.
- If one action is weakly dominant, the rest are **weakly dominated**.

Because only one player has a strictly dominant strategy, this game doesn't have a dominant strategy solution. But we can predict what will happen by using **iterated elimination of strictly dominated strategies**.

- Bicyclist will never play Focus. (Focus is strictly dominated.)
- Since Professor knows this, Professor never plays Road.
  (Once Focus is crossed out, Road is strictly dominated.)
- By iterated elimination, the solution is (Sidewalk, Text).

"Iterated" elimination just means "step-by-step": we go back and forth between players, crossing out strategies that are now strictly dominated.

Example 3: the Cuban Missile Crisis

Now let's consider a very serious game indeed: the Cuban Missile Crisis.

- 1962: Soviet Union installs nuclear missiles in Cuba.
- US and Soviets decide whether to "Attack" or "Negotiate".
- If one Attacks while the other is still trying to Negotiate, whoever chose to Attack has a military advantage.[2]

Payoff matrix:

		Soviets: Attack	Soviets: Negotiate
US	Attack	$-2, -3$	$0, -5$
US	Negotiate	$-4, -1$	$2, 2$

Let's look for strictly dominant (and dominated) strategies:

- Does the US have a strictly dominant strategy? No. If it expects the Soviets to Attack, the US should also Attack. If it expects the Soviets to Negotiate, the US should also Negotiate.
- Does the Soviet Union have a strictly dominant strategy? No. The Soviets face incentives similar to those the US faces.

In fact, there aren't even any weakly dominant strategies. So we can't rule anything out! Depending on what each country expects the other to do, the world may either slip into a disastrous war or remain at peace.

Game theory gives us a useful lens for understanding why the Cuban Missile Crisis was such a dangerous moment in world history—there were **multiple equilibria**, either of which could have happened. Nobody *wants* war here, but a war might still occur if both countries *expect* it.

[2] Furthermore, all else equal, the US had a slightly stronger military in 1962. The payoffs I've chosen for this problem reflect the military imbalance between the two countries.

Best responses and Nash equilibrium

A strategy is a **best response** if it maximizes a player's payoff given what they expect the other player to do.

- In the Missile Crisis game, the best response to "Attack" is "Attack".
- You can have multiple best responses if they're "tied for first place". Both "Sidewalk" and "Road" are best responses to "Focus".

What's the relationship between a best response and a dominant strategy?

- A strictly domin<u>ant</u> strategy is a best response to *every* opposing strategy. A strictly domin<u>ated</u> strategy is *never* a best response.
- A weakly dominant strategy is always a best response, too, but there's at least one situation where it's only "tied for first".
- Not every best response is a dominant strategy. In the Missile Crisis, "Attack" is a best response to "Attack", but not to "Negotiate".

A pair of pure strategies is a **pure strategy Nash equilibrium** (PSNE) if each player's strategy is a best response to the other's strategy.

- A Nash equilibrium is a <u>self-fulfilling prophecy</u>: if we both expect a particular Nash equilibrium to happen (e.g., war between the US and the Soviet Union), our expectations will *cause* it to happen.
- A Nash equilibrium is <u>self-enforcing</u>: if the players could make an agreement in advance to play a certain Nash equilibrium, neither player would have any incentive to break their agreement.
- A Nash equilibrium <u>never causes regret</u>: if both players follow the script, neither regrets her action after seeing what the other one did.

For these reasons, we typically predict that the players will end up playing a Nash equilibrium. This is the main solution concept used in game theory. (It's named after John Nash, who won the 1994 Nobel Prize in Economics.)

Zero, one, or many?

A game can have zero, one, or many pure strategy Nash equilibria.

- Prisoner's Dilemma has one PSNE (Flip, Flip).
- So does the bike-accident game (Sidewalk, Text).
- The Cuban Missile Crisis game has two PSNEs.
- "Name the Biggest Number" has zero PSNEs.

You will get a lot of practice finding PSNEs.

How to find all pure strategy Nash equilibria (if any)

We can actually use the same procedure to solve all of the games above:

1. Find each player's best response to each of the other one's strategies.
2. Look for combinations where both players are best-responding.
3. Any such combination is a PSNE. If none exists, there are no PSNE.

Since every dominant strategy is a best response, this procedure detects dominant strategy solutions as well as all other Nash equilibria.

In practice, a simple way to find these combinations is to circle each payoff corresponding to a player's best response. Two circles indicate a PSNE:

		Clyde: Flip	Clyde: Deny
Bonnie	Flip	0, 0	10, −2
	Deny	−2, 10	9, 9

		Bicyclist: Focus	Bicyclist: Text
Prof.	Sidewalk	1, 2	1, 5
	Road	1, 2	−5, 3

The term "Nash equilibrium" always refers to the strategies, not to the payoffs the players receive: "(Flip, Flip)" is a PSNE. "(0, 0)" is not a Nash equilibrium: instead, we call "(0, 0)" the **equilibrium payoffs**.

