Lecture Note 14: Dealing with Risk

People are exposed to all kinds of financial and non-financial risks:

- You might get into a car accident.
- Your house might be destroyed by a hurricane.
- If a company goes bankrupt, its stock becomes worthless.

How can risk-averse agents reduce their exposure to risk?

1. Avoidance: reducing the likelihood that an adverse event occurs (or reducing the negative consequences experienced if it does occur).
   - Store valuables in a fire-proof safe.
   - Don't buy a house in a flood zone.
2. Diversification: placing many small bets instead of one big one.
   - You could invest in an index fund (not just Enron stock).
   - A company can own property in several cities (not just Miami).
3. Insurance: receiving compensation when an adverse event occurs.
   - Health insurance.
   - Malpractice insurance.
4. Information: in the presence of uncertainty, people may have an incentive to gather information that can improve their decisions.
   - Research a financial asset before buying it.
   - Date somebody before marrying them.

We'll study 1–3 in class. HW#7 will also explore the value of information.

Approach #1: Avoidance

Consider an agent who has a utility function

$$u(w,h) = w + 10\sqrt{h}$$

where w is her wealth and h is her health. She starts with wealth $w_0 = 20$ and health $h_0 = 100$, but there is a probability $\lambda = 10\%$ that she contracts a dangerous disease that reduces her health by $d = 51$ (so that $h = 49$).

How much would she be willing to pay (i.e., how much wealth would she be willing to give up) for a vaccine that eliminates the risk of this disease?

Without the vaccine, the agent's expected utility is

$$\begin{aligned}
\text{EU(no vaccine)} &= \underbrace{\lambda u(w_0, h_0 - d)}_{\text{I might get sick}} + \underbrace{(1-\lambda)u(w_0, h_0)}_{\text{I might not get sick}} \\
&= 0.1 \times (20 + 10\sqrt{49}) + 0.9 \times (20 + 10\sqrt{100}) \\
&= 117
\end{aligned}$$

If the agent pays x to buy the vaccine, her expected utility is

$$\begin{aligned}
\text{EU(vaccine)} &= \underbrace{u(w_0 - x, h_0)}_{\text{vaccine} \implies \text{never get sick}} \\
&= 20 - x + 10\sqrt{100} \\
&= 120 - x \qquad (1)
\end{aligned}$$

The agent's willingness to pay for the vaccine is the value x^* that makes her *indifferent*. It is implicitly defined by the **indifference condition**

$$\begin{aligned}
\text{EU(no vaccine)} &= \text{EU(vaccine)} \\
117 &= 120 - x
\end{aligned}$$

So, the agent is willing to pay $x^* = \$3$ for the vaccine.

Approach #2: Diversification

There's an old saying that you shouldn't "put all your eggs in one basket": if you drop the basket, all the eggs get broken. Better to spread them out.

Suppose I'm deciding whether to invest in Vandelay Industries or Gobias. A share of either stock is worth \$20 with probability $\frac{1}{4}$ and it's worth \$0 with probability $\frac{3}{4}$. Let X_V and X_G denote each stock's share price.

If I buy an **undiversified portfolio** consisting of two shares of Vandelay, then I have a $\frac{1}{4}$ chance of getting \$40 and a $\frac{3}{4}$ chance of getting \$0.

We can assess this portfolio by computing its mean and variance:

$$\mathbb{E}(X_V + X_V) = \frac{1}{4}(40) + \frac{3}{4}(0) = 10$$
$$\text{Var}(X_V + X_V) = \frac{1}{4}(40 - 10)^2 + \frac{3}{4}(0 - 10)^2 = 300$$

What if I instead buy a **diversified portfolio** consisting of one share of each stock? If the stocks are *uncorrelated* with each other,

- There's a $\frac{1}{16}$ chance both are worth \$20 (because $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$).
- There's a $\frac{9}{16}$ chance both are worth \$0 (because $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$).
- There's a $\frac{6}{16}$ chance that one is worth \$20 and one is worth \$0.

Therefore, this diversified portfolio has

$$\mathbb{E}(X_V + X_G) = \frac{1}{16}(40) + \frac{6}{16}(20) + \frac{9}{16}(0) = 10$$
$$\text{Var}(X_V + X_G) = \frac{1}{16}(40 - 10)^2 + \frac{6}{16}(20 - 10)^2 + \frac{9}{16}(0 - 10)^2 = 150$$

The diversified portfolio is just as good on average (same mean), but it's less risky than the undiversified portfolio (lower variance).

Idiosyncratic vs. aggregate risk

The extent to which diversification reduces risk depends on the extent to which the prices of these stocks are **correlated** with each other.

- If Vandelay and Gobias are **perfectly positively correlated** (either both succeed or both fail), then a diversified portfolio is just as risky as investing in a single company.
- If Vandelay and Gobias are **perfectly negatively correlated** (one will succeed, one fails), then diversification eliminates the risk.
- If Vandelay and Gobias are **uncorrelated** with each other, then diversification reduces the risk but doesn't eliminate it.

Diversification works by protecting you against the **idiosyncratic risk** that a particular asset will perform badly.

- There could be a scandal involving Vandelay's CEO.
- There could be a defect in one of Gobias's leading products.

In fact, by investing in *lots* of companies—not just two—investors can essentially eliminate this kind of risk, thanks to the law of large numbers.

But even a diversified stock portfolio is vulnerable to the **aggregate risk** that the stock market will perform badly overall.

- A trade war between the US and China could trigger a recession.
- Congress could pass a law raising the tax on dividend income.

Another example: a commercial landlord could reduce its exposure to hurricane risk by owning property throughout the country, but it will still be vulnerable to nationwide changes in demand for office space.

Approach #3: Insurance

An **insurance contract** is a legal arrangement under which:

- The policyholder pays a fee (the **premium**) to the insurer
- If a qualifying event occurs (e.g., a car crash), the policyholder is entitled to receive financial compensation from the insurer.

Imagine that you've just bought a \$500K house in Key West, Florida, and that you also have \$100K in the bank. There is a 1% chance that your home will be destroyed by a hurricane within the next year.

- In the event of a hurricane, your wealth is \$100K.
- Otherwise, it's \$600K.

A **full insurance** policy fully compensates the policyholder for her loss. If the premium is p, a homeowner will purchase the insurance policy if utility under full insurance exceeds the expected utility without insurance:

$$u(600 - p) \geq .01u(100) + .99u(600)$$

The **expected claim** is the expected value of the compensation received from the insurance company. Under full insurance,

$$\text{expected claim} = 1\% \cdot \$500K + 99\% \cdot \$0 = \$5000$$

An insurance contract is said to be **actuarially fair** (or simply "fair") if the premium equals the expected claim. In other words, an actuarially fair insurance policy "breaks even" on average.

A risk-averse agent will always want to purchase actuarially fair insurance.*

*Thanks to a mathematical result called *Jensen's inequality*, the condition

$$u(595) \geq .01u(100) + .99u(600)$$

holds if and only if $u(\cdot)$ is a concave function, i.e., it holds if and only if the agent is risk-averse.

Risk attitudes and insurance decisions

Thinking about insurance is a great way to check our understanding of risk attitudes more generally.

At actuarially fair prices,

- Risk-averse agents always want to buy insurance.
- Risk-neutral agents are indifferent about buying insurance.
- Risk-loving agents never want to buy insurance.

In practice, insurance is never actuarially fair:

- Insurers have to cover administrative overhead (e.g., salaries paid to salespeople and actuaries) as well as the cost of claims.
- Insurers may have market power, enabling them to charge a markup above the marginal cost of providing insurance.

Suppose that hurricane insurance is more expensive than actuarially fair.

- Risk-averse agents might buy insurance, but might not.
- Risk-neutral agents never want to buy insurance.
- Risk-loving agents never want to buy insurance.

