Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

TA Section Problems #1
(Week of Monday, September 30)

1 Tax and tip

Throughout this problem, "price" means the amount paid by a consumer to a barber for a haircut itself, not including any taxes or tips that the consumer pays in addition to the price.

New York City has a perfectly competitive market for haircuts, with demand for haircuts given by $p_D(Q) = 30 - 2Q$ and supply of haircuts given by $p_S(Q) = 12 + Q$.

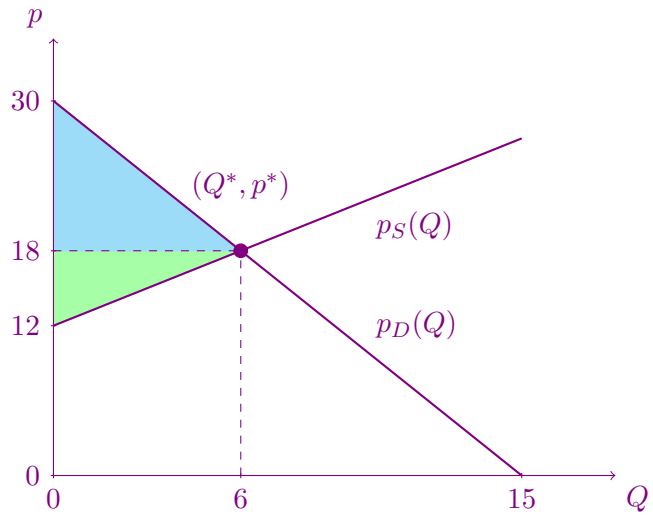
New Yorkers (like me) have a reputation for being rude, so let's assume that nobody tips their barber unless they are legally obligated to do so.

a. Compute the equilibrium price and quantity.

   In equilibrium, supply must equal demand: $30 - 2Q = 12 + Q \implies Q^* = 6, p^* = 18$.

b. Compute the consumer surplus, producer surplus, and deadweight loss.

   It's helpful to draw the graph here:



The consumer surplus is $\frac{1}{2}(30-18)(6) = 36$. Producer surplus is $\frac{1}{2}(18-12)(6) = 18$. As usual in a competitive market with no distortions, there is no deadweight loss here.

c. Suppose that the city council imposes a \$6 tax on haircuts, paid by the consumer. Find the new equilibrium price and quantity. Is there any deadweight loss?

Since the tax is imposed on consumers, the demand curve shifts down by \$6. Why? Remember that the demand curve represents the most that a given consumer is willing to pay the seller for a unit of the good. If consumers know they will have to write a \$6 check to the city after the transaction is complete, the amount they're willing to pay the seller will fall by \$6. The new demand curve is

$$p_D(Q) = 30 - 2Q - 6 = 24 - 2Q$$

Since producers don't (directly) pay the tax, the supply curve is unchanged:

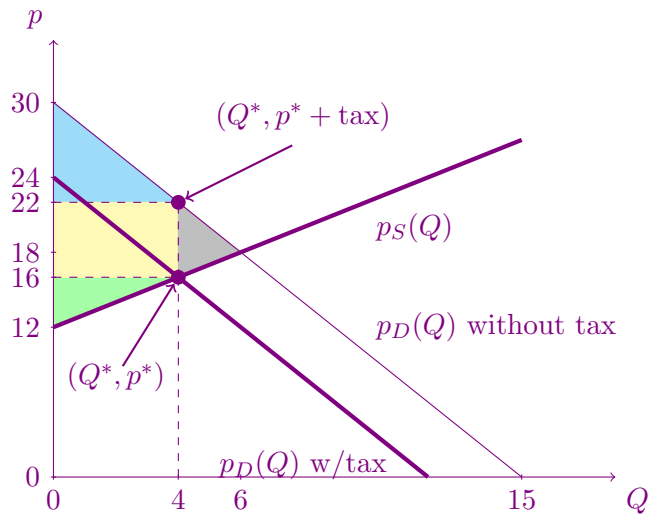
$$p_S(Q) = 12 + Q$$

To find the new equilibrium, we find the point where the supply and demand curves intersect:

$$24 - 2Q = 12 + Q \implies Q^* = 4 \implies p^* = 16$$

Both the price charged by barbers and the number of haircuts decrease as a result of the tax. Both buyers and sellers are worse off than before: consumers have to pay a total of \$22 instead of \$18 for each haircut (\$16 to the barber shop, \$6 to the city), and barber shops only receive \$16 instead of \$18 for each haircut they sell.

Here is a graph showing the new equilibrium:



I've shaded in the consumer surplus (blue), producer surplus (green), tax revenue (yellow), and deadweight loss (gray). Comparing this graph to the previous one, we

can see that both producer and consumer surplus have decreased. Furthermore, there is deadweight loss now, equal to $\frac{1}{2}(22-16)(6-4)=6$. The DWL represents the "missed opportunities" from socially desirable transactions that do not take place: between $Q=4$ and $Q=6$, consumers' willingness to pay exceeds suppliers' marginal cost of production, but these potential "gains from trade" sadly go unexploited.

d. In an effort to improve New York's bad reputation, the city council replaces the tax with a law requiring consumers to give their barber a \$6 tip after a haircut. New York residents grumble and complain (we like to complain), but they obey the new law.

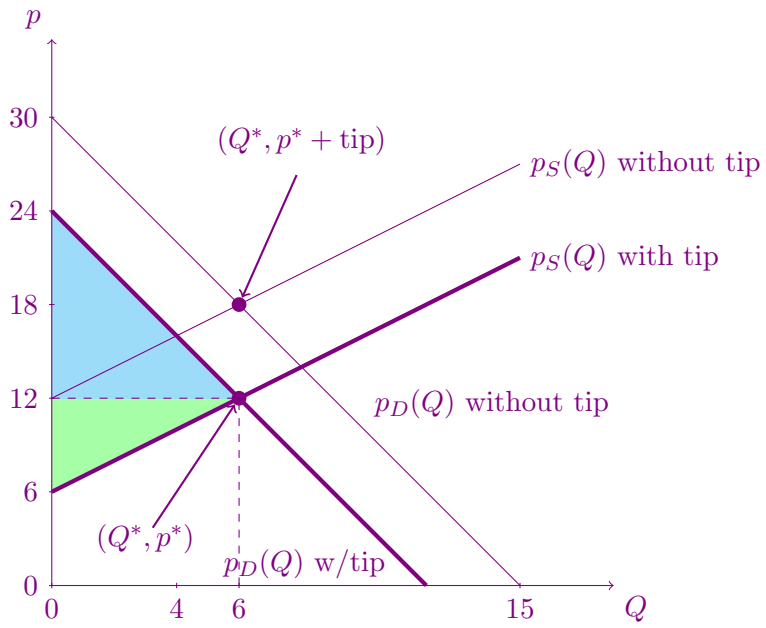
Find the new equilibrium price and quantity. Is there any deadweight loss?
(Hint: Will the demand curve shift? What about the supply curve?)

From the perspective of consumers, the tipping law is just like a \$6 tax, so the demand curve once again shifts down by \$6. From the perspective of barbers, the tipping law is really like a \$6 *subsidy*, since—for any given price—suppliers receive an extra \$6 for each haircut they sell. As a result, the supply curve shifts down by \$6, as well:

$$p_D(Q) = 24 - 2Q$$
$$p_S(Q) = 6 + Q$$

Setting these terms equal and solving for Q, we find that $Q^* = 6$ and $p^* = 12$.

We can see all of this in the graph below:



So, what is going on here? The market price has adjusted to fully offset the impact of the mandatory tipping law: instead of paying barbers \$18 for a haircut, consumers now pay a regular price of \$12 plus a \$6 tip—which is essentially the same thing.

The law has no impact on the number of haircuts sold, the amount of money paid by consumers, or the amount of money received by barbers. Relative to the unregulated equilibrium, there is no change in consumer or producer surplus, and there is once again no deadweight loss: all haircuts that *should* occur *do* occur.

2 Tooling up

A soybean farmer can sell as many units of soybeans as she wants at a price $p = 20$. With her current farm equipment, her cost of producing q units of soybeans is $C(q) = 2q^2 + 4q$. She is deciding whether to buy a new tractor, which would lower her production costs to $C(q) = q^2 + 2q$. How much would the farmer be willing to pay for the tractor?

Without the tractor, the farmer solves the profit-maximization problem

$$\max_q \ 20q - 2q^2 - 4q \quad \text{or equivalently} \quad \max_q \ 16q - 2q^2$$

The FOC is $16 - 4q^* = 0 \implies q^* = 4$. The profit is $\pi(q^*) = 20(4) - 2(4)^2 - 4(4) = 32$.

If she buys the tractor, her new profit-maximization problem is

$$\max_q \ 20q - q^2 - 2q - F \quad \text{or equivalently} \quad 18q - q^2 - F$$

where F is the cost of the tractor. Since F is a fixed cost, we treat it as a constant in the maximization problem, so the FOC is simply $18 - 2q = 0 \implies q^* = 9$. In this case, the farmer's profits (after deducting the cost of the tractor) are $\pi(q^*) = 18(9) - 9^2 - F = 81 - F$.

To determine the farmer's willingness to pay for the tractor, we have to find the value of F that makes the farmer indifferent between buying or not buying the tractor. The farmer is indifferent when she makes the same profit either way, so this special value F^* is given by

$$32 = 81 - F^* \implies F^* = 49$$

