Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

TA Section Problems #2
(Week of Monday, October 7)

Your friendly neighborhood monopolist

Demand for internet in Dixon, CA is given by $p(Q) = 120 - \frac{1}{10}Q$. Comcast can supply Q households with internet access at a total cost of $C(Q) = 20Q$.

a. If Comcast sets the price of internet as though it were a competitive firm, what would it charge for internet (p_c)? How many households would get internet (Q_c)?

If Comcast sets a competitive price, the price will equal marginal cost:

$$p(Q) = 120 - \frac{1}{10}Q = 20 \implies Q_c = 1000 \text{ and } p_c = 20$$

b. If Comcast engages in uniform monopoly pricing, what price will it charge (p_m)? How many households will get internet (Q_m)? Compute the deadweight loss.

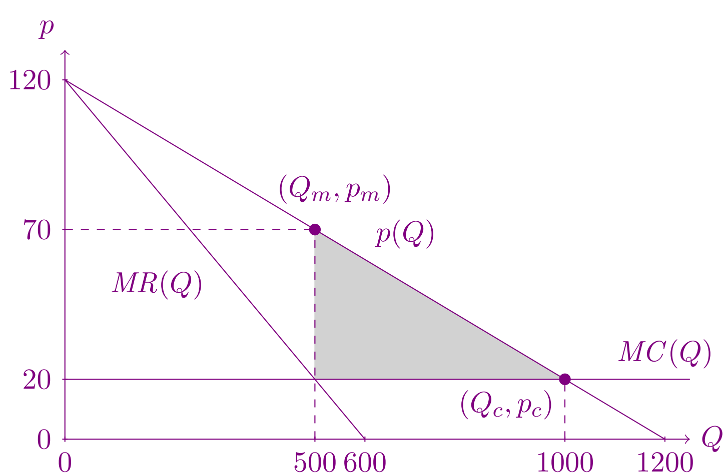
Comcast solves the profit-maximization problem

$$\max_Q \; \pi(Q) = p(Q)Q - C(Q) \implies \max_Q \; \left(120 - \frac{1}{10}Q\right)Q - 20Q$$

Taking the first-order condition (or, equivalently, setting $MR = MC$), we get

$$120 - \frac{1}{5}Q - 20 = 0 \implies Q_m = 500, \; p_m = 70$$

The deadweight loss (shown in gray below) equals $\frac{1}{2}(1000 - 500)(70 - 20) = 12{,}500$:



c. Governments often regulate monopolies by setting price ceilings that limit what they may legally charge. How many households will get internet if Dixon sets

   i. a price ceiling equal to 100?
   ii. a price ceiling equal to 19.99?
   iii. a price ceiling equal to 20.01? (An approximate answer is fine here.)
   iv. a price ceiling equal to 60?

If the price ceiling is set to 100, it doesn't bind: the monopoly will charge a price $p_m = 70$ and sell $Q_m = 500$ as before, since its original price already complies with the price ceiling.

If the price ceiling is set to 19.99, Comcast would make negative profit on any units it sold. So, Comcast sets $Q_m = 0$ and no households get internet.

If the price ceiling is set to 20.01, the price ceiling is "binding" and Comcast charges the highest price it can: $p_m = 20.01$. At this price, Comcast makes positive profit on every unit it sells, and so it sells internet to every household that demands internet at that price. Since $p_m = 20.01$ is very close to the competitive price, $p_c = 20$, the quantity sold is approximately equal to the quantity sold at the competitive price, $Q_c = 1000$. (For an exact solution, we can invert the demand curve to obtain $Q(p) = 1200 - 10p$, which means $Q_m = 999.9$.)

If the price ceiling is set to 60, the price ceiling is once again binding, and Comcast charges $p_m = 60$. At this price, the quantity demanded is $Q_m = 1200 - 10 \cdot 60 = 600$. In this case, Comcast sells an amount below what it would sell under perfect competition, but above what it would sell as an unregulated monopoly.

For the price ceilings at 20.01 and at 60, how can be sure that the price ceiling will bind? Let's compute Comcast's profit as a function of price, again using $Q(p) = 1200 - 10p$:

$$\pi(p) = pQ(p) - C(Q(p)) = p(1200 - 10p) - 20(1200 - 10p) = 1400p - 10p^2$$

Comcast's marginal profit is therefore

$$\frac{d\pi}{dp} = 1400 - 20p$$

This is a positive number as long as $p < 70$, which means that—for any price ceiling set below 70—Comcast's profit-maximizing choice is to charge the highest price it's allowed to charge and to sell to anyone who wants to buy at that price.

Bowling alone

An entrepreneur is deciding whether to open a bowling alley in a small town. Building the alley requires a fixed cost FC up front. If she builds the alley, the entrepreneur faces demand $p(Q) = 18 - 2Q$ and has a constant marginal cost of 2. If she doesn't build it, she gets 0.

a. Suppose that the entrepreneur would be a uniform-pricing monopolist. If she opens the alley, how much revenue will she make? For what value of FC is she indifferent about opening it?

If the entrepreneur opens the bowling alley, her profit is

$$\max_Q \, (18 - 2Q)Q - 2Q - FC$$

The FOC is $18 - 4Q - 2 = 0$, which implies $Q_m = 4$ and hence $p_m = 10$. Total revenue equals $p_m \cdot Q_m = 4 \cdot 10 = 40$. Plugging Q_m back into the profit function gives $\pi = 32 - FC$. The entrepreneur is indifferent if $FC = 32$, since in that case she makes the same (zero) profit regardless of what she does.

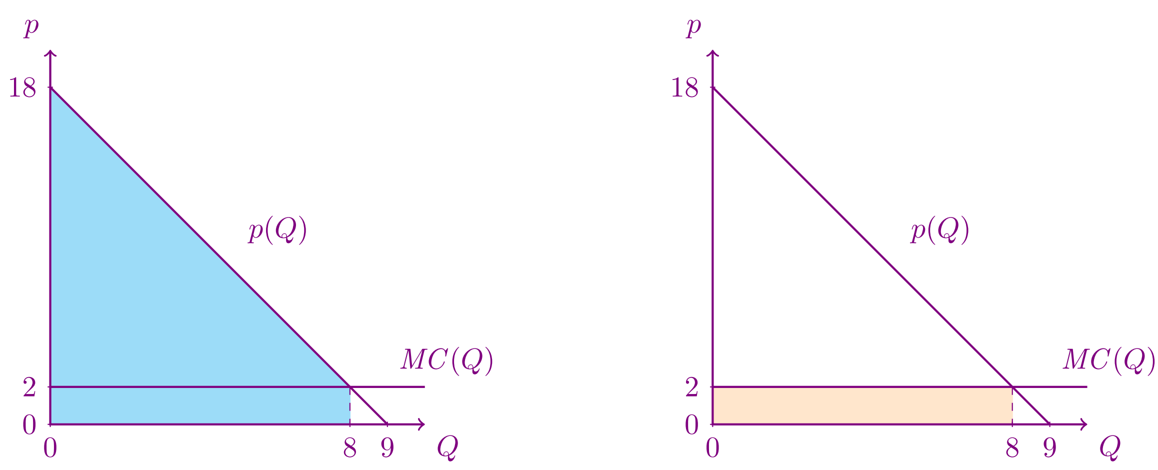
b. Now suppose she can engage in perfect price discrimination. If she opens the alley, how much revenue will she make? For what value of FC is she indifferent about opening it?

Let's graph the market. Under perfect price discrimination, the entrepreneur will sell $Q = 8$ units, since for all of these transactions the consumer's willingness to pay (WTP) exceeds the marginal cost of production. Each of these consumers will be charged an individualized price equal to his WTP. Total revenue is therefore the area under the demand curve, from $Q = 0$ to $Q = 8$. It is shown in blue in the lefthand panel below.

To produce these units, the entrepreneur must pay variable costs equal to the area under the marginal cost curve from $Q = 0$ to $Q = 8$, as well as the fixed cost FC. The variable costs are shown in orange in the righthand panel below.

Revenue under PPD

Variable costs under PPD



Using the graph, we can calculate total revenue by adding the size of the blue triangle (area = 64) and the size of the blue rectangle (area = 16). Therefore, total revenue is 80. The variable costs of production are $2 \cdot 8 = 16$. So, the entrepreneur's profit equals

$$\pi = 80 - 16 - FC = 64 - FC$$

She is indifferent about opening the bowling alley if $FC = 64$, since in that case $\pi = 0$. Notice that the entrepreneur is willing to pay more up front if she knows she'll be able to price discriminate, since price discrimination makes the bowling alley more profitable.

Zoom zoom zoom

Demand for electric scooters is given by $p(Q) = \sqrt{16 - Q}$.

a. Calculate the price elasticity of demand as a function of Q.

   Rewrite the demand curve as $Q(p) = 16 - p^2$. Then

$$\varepsilon = \frac{dQ}{dp}\frac{p}{Q} = (-2p)\frac{p}{Q} = -\frac{2p^2}{Q} = -\frac{2(16-Q)}{Q} = 2 - \frac{32}{Q}$$

b. Compute the elasticity when $Q = 0$ and when $Q = 16$. What terms do we use to describe these elasticities? At what quantity is demand unit elastic?

   Using the formula we found above:

   i. $Q = 0 \implies \varepsilon = -\infty$ (perfectly elastic)
   ii. $Q = 16 \implies \varepsilon = 0$ (perfectly inelastic)
   iii. Demand is unit elastic when $\varepsilon = -1$, which occurs when $Q = \frac{32}{3}$.