Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Section Problems #3
(Week of Monday, October 14)

Living on the edge

Skyfall, Inc. is a monopoly supplier of skydiving lessons in a remote desert town. Skyfall faces demand $p_L(Q_L) = 24 - Q_L$ from locals and demand $p_T(Q_T) = 36 - Q_T$ from tourists. Skyfall faces costs $C(Q_L, Q_T) = 12Q_L + 12Q_T$.

a. Suppose that Skyfall can engage in group price discrimination (it's easy to tell who's a tourist). Find Q_L^* and Q_T^*. Then find the prices p_L^* and p_T^* as well as Skyfall's profits.

The profit maximization problem is

$$\max_{Q_L, Q_T} \pi(Q_L, Q_T) = (24 - Q_L)Q_L + (36 - Q_T)Q_T - 12Q_L - 12Q_T$$

Assuming that there is an interior solution for both quantities, the FOCs give

$$24 - 2Q_L - 12 = 0 \implies Q_L^* = 6 \implies p_L^* = 18$$

and

$$36 - 2Q_T - 12 = 0 \implies Q_T^* = 12 \implies p_T^* = 24$$

Since both quantities are non-negative, these candidate solutions are indeed optimal. Skyfall's profits are 180. (It makes profit of 36 from sales to locals, and an additional 144 from profits to tourists.)

A travel agency sues Skyfall for unfairly discriminating against tourists. As part of a legal settlement, Skyfall agrees to charge the same price for all customers.

b. Calculate the market demand curve $Q(p)$. (Hint: you will need to describe this curve "piecewise", using one equation for $p < 24$ and another for $p \geq 24$.)

For $p \geq 24$, only tourists have positive demand, so the demand curve is $p(Q) = 36 - Q$, which we can invert to get $Q(p) = 36 - p$. For $p < 24$, both tourists and locals have positive demand. To get market demand, we first invert each group's demand curve:

$$Q_L(p) = 24 - p$$
$$Q_T(p) = 36 - p$$

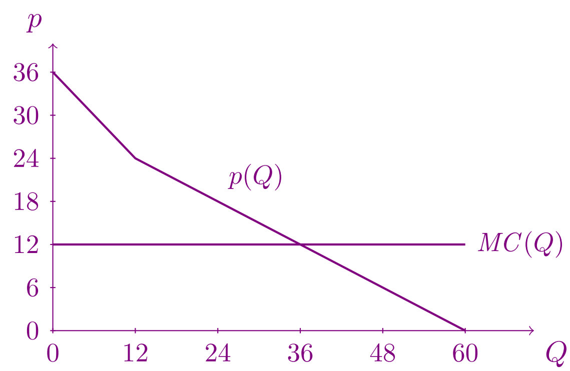
Adding these curves together gives us $Q(p) = Q_L(p) + Q_T(p) = 60 - 2p$. Putting it all together, we get the market demand curve

$$Q(p) = \begin{cases} 60 - 2p & \text{for} \quad p < 24 \\ 36 - p & \text{for} \quad p \geq 24 \end{cases}$$

If we want to graph this function, we can invert it again to get

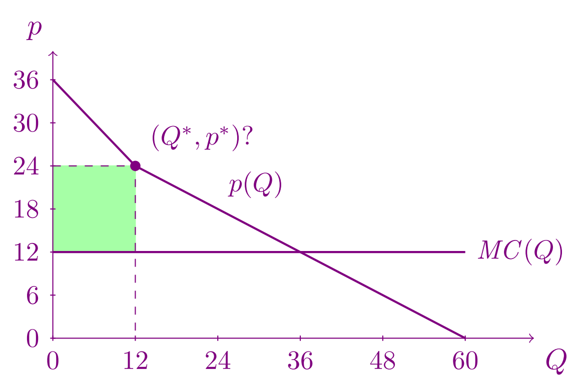
$$p(Q) = \begin{cases} 36 - Q & \text{for} \quad Q \leq 12 \\ 30 - \frac{1}{2}Q & \text{for} \quad Q \leq 24 \end{cases}$$

The market demand curve looks like this



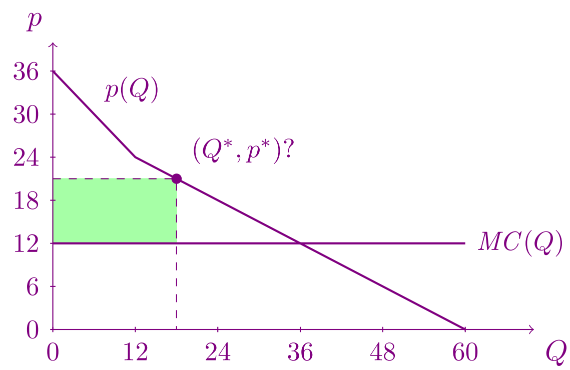
c. Suppose that Skyfall decides to set a price $p \geq 24$ (high enough that no locals buy). If Skyfall pursues this strategy, what price will it charge, and what are its profits?

If Skyfall charges $p \geq 24$, it faces the demand curve $p(Q) = 36 - Q$, which is identical to the tourist demand it faced in part a. In this case, Skyfall's best option is to set a price $p^* = 24$. (Note that, at this price, $Q_L^* = 0$: no locals are interested in buying unless the price drops below 24.) Skyfall would sell $Q^* = 12$, and its profits would therefore be: $\pi = 24 \cdot 12 - 12 \cdot 12 = 144$. This candidate solution is plotted below.



d. Now suppose that Skyfall sets a price $p < 24$ (low enough that both groups buy). Under this strategy, what price will Skyfall charge, and what are its profits?

Under this strategy, Skyfall faces market demand $p(Q) = 30 - \frac{1}{2}Q$. Maximizing profits gives us the solution $p^* = 21$, which corresponds to the quantity $Q^* = 18$. Under this strategy, profits are $\pi = 21 \cdot 18 - 12 \cdot 18 = 162$. This candidate solution is shown below.



e. Compare Skyfall's profits under the "candidate" prices you identified in parts c and d. What will Skyfall charge? Who is better off under uniform monopoly pricing than under group price discrimination? Who is worse off?

Skyfalls profits are higher under the candidate price $p^* = 21$ we found in part d, so this is the price it will charge. Relative to group price discrimination, tourists face a lower price, so they are better off. Locals, however, face a higher price than they did before, so they are worse off. Skyfall is worse off too: under group price discrimination, its profits were 180, and they have now fallen to 162.

This is no surprise: under group price discrimination, Skyfall *could* have chosen to set a price of $p^* = 21$ for both groups. The fact that it chose to price discriminate indicates that it could make additional profit by doing so.

Living in the moment

A bakery makes bread according to the Cobb-Douglas production function $q(L, K) = \sqrt{LK}$. It sells loaves of bread in a competitive product market at price $p = 4$. In the short run, the bakery's capital stock is fixed at $K = 200$, so that its production function simplifies to

$$q(L) = \sqrt{200L}$$

The bakery hires workers in a competitive labor market, where the market wage is 10.

a. Compute the bakery's optimal (short-run) choice of labor, L^*. Then compute its (short-run) physical output q^*.

The bakery chooses labor to solve

$$\max_L \; 4\sqrt{200L} - 10L$$

I've ignored the rental cost of capital: since we're looking at the short run, with K fixed, the capital cost rK won't affect the bakery's choice (it's constant). The FOC is

$$\frac{4\sqrt{200}}{2\sqrt{L}} - 10 = 0 \implies L^* = 8$$

Plugging this back into the production function yields

$$q(L^*) = \sqrt{200 \times 8} = \sqrt{1600} = 40$$

b. What is the *marginal physical product of labor* at the optimum (when $L = L^*$)? What is the *marginal revenue product of labor* at the optimum?

At the optimum, the marginal revenue product of labor equals the wage: $MRPL = 10$. And we know that $MRPL = MR \times MPPL$. Since the bakery is a price-taker, its marginal revenue is just the price. Rearranging gives

$$MPPL = \frac{MRPL}{p} = \frac{10}{4} = 2.5$$

We could also directly solve $MPPL(L^*) = q'(L^*) = \frac{\sqrt{200}}{2\times\sqrt{8}} = 2.5$.