Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Section Problems #4
(Week of Monday, October 28)

Practice makes perfect

For each of the games below:

- Circle all payoffs corresponding to a player's best response.
- Indicate whether each player has a *strictly* dominant strategy. If so, which strategy?
- Identify any strategies that are *weakly* dominant but not *strictly dominant*.
- Identify any/all pure strategy Nash equilibria (PSNEs)—remember to write each PSNE as an ordered pair of strategies—as well as the corresponding equilibrium payoffs.

a. **Lord of the Rings**: Mordor's troops are ready to invade Gondor ...

		Gondor	
		Freak Out	Chill Out
Mordor	Attack	(5), (0)	(10), −1
	Ignore	0, 5	0, (8)

Mordor has a strictly dominant strategy, Attack. Gondor has no strictly dominant strategy. None of the strategies are weakly dominant but not strictly dominant. There is one PSNE: (Attack, Freak Out), with equilibrium payoffs (5, 0).

b. **Looney Tunes**: Coyote is hungry: where should he look for the Roadrunner?

		Roadrunner	
		River	Cliff
Coyote	River	(10), −2	0, (5)
	Cliff	0, (5)	(10), −2

There are no strictly dominant strategies, and there are also no weakly dominant strategies. There is no PSNE. (Here is some intuition for what is going on in this problem. Coyote is trying to catch Roadrunner, and Roadrunner is trying to avoid Coyote. So, whatever choices they make, one of them will "guess wrong" and wish that they could pick a different strategy once they learn what the other player chose.

Therefore, every pair of strategies results in somebody feeling "regret"—which means there cannot be a PSNE.)

c. **Price competition**: What kind of price should each coffee shop charge?

		Starbucks		
		Low	Med	High
Peet's	Low	(2), (2)	3, 1	4, 0
	Med	(2), 3	(4), (4)	(7), 2
	High	(2), 4	2, 5	5, (6)

There are no strictly dominant strategies. For Peet's, the strategy Med is weakly dominant but not strictly dominant: it never yields a *worse* payoff than Low or High, and if Starbucks plays Med or High, then playing Med gives Peet's a *better* payoff than its alternative strategies. There are two PSNEs: (Low, Low), with corresponding equilibrium payoffs (2, 2), and (Med, Med), with equilibrium payoffs (4, 4).

d. **The timing of retirement**: When should each partner choose to retire? (Note: it's perhaps a little unrealistic to think of this as a static game. But let's solve it anyway.)

		Quinn		
		Age 60	Age 65	Age 70
Alex	Age 60	1, 1	1, (3)	0, 1
	Age 65	2, 3	5, (12)	6, −3
	Age 70	(3), 4	(6), (5)	(8), −1

Alex has a strictly dominant strategy: Age 70. So does Quinn: Age 65. None of the strategies are weakly dominant but not strictly dominant. There is one PSNE: (Age 70, Age 65), with equilibrium payoffs (6, 5). Notice that, since each player has a strictly dominant strategy, this PSNE is also a dominant strategy solution.

Desperadoes, incommunicado

The year is 1902. Butch Cassidy and the Sundance Kid (two famous bandits) have made plans to get together and rob something tonight, but they forgot to decide whether they're going to rob a train or a bank. Cell phones haven't been invented yet, so each of these two players must decide where to go without knowing what the other one will do.

The numbers below indicate how much money each bandit makes from their robbery attempt. Since they each move once, move at the same time, and are ignorant of each other's action when making their own choice, this problem starts out as a static game.

		The Kid	
		Train	Bank
Cassidy	Train	(7), (3)	0, 2
	Bank	2, 0	(3), (5)

a. Circle all payoffs corresponding to best responses. Find any/all PSNEs.

See circled payoffs above. The PSNEs are (Train, Train) and (Bank, Bank).

When the outcome is uncertain, each player's ***expected payoff*** *is a weighted average of his possible payoffs, where the weights are the probability associated with each possible payoff. For example, if I have a 10% chance of getting $200 and a 90% chance of getting $30, my expected payoff is* $10\% \times \$200 + 90\% \times \$30 = \$20 + \$27 = \$47$.

b. Suppose that Cassidy thinks there's a 50% chance The Kid will pick the train and a 50% chance The Kid will pick the bank. What is Cassidy's expected payoff from each of his two strategies? Which one will he pick? If p is the probability that The Kid picks the train, for what value p^* is Cassidy indifferent between the two strategies?

With 50-50 probabilities, Cassidy's expected payoffs are:

- Train: expected payoff is $(.5)(7) + (.5)(0) = 3.5$.
- Bank: expected payoff is $(.5)(2) + (.5)(3) = 2.5$.

Since the train has a bigger expected payoff, Cassidy will pick the train.

Now suppose that Cassidy thinks there's a probability p The Kid will pick the train and a probability $1 - p$ The Kid will pick the bank. Cassidy's expected payoffs are as follows:

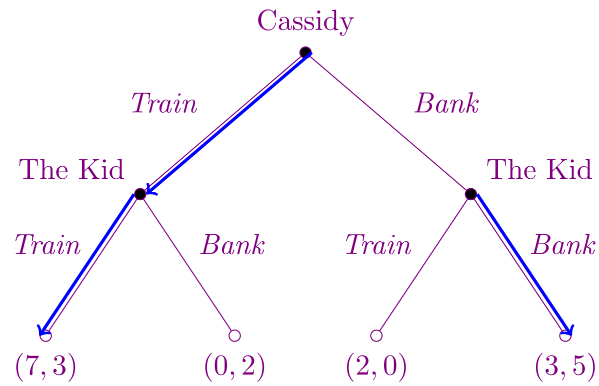
- Train: expected payoff is $(p)(7) + (1 - p)(0) = 7p$.
- Bank: expected payoff is $(p)(2) + (1 - p)(3) = 3 - p$.

Cassidy is indifferent when these expected payoffs are equal: $7p = 3 - p \implies p^* = \frac{3}{8}$.

The next few parts are about dynamic *games, where one player moves before the other. Here, you can think of the player who sends the message as moving first. I expect that we will start covering dynamic games on Tuesday 10/29 and finish on Thursday 10/31.*

c. Now suppose that Cassidy can send The Kid a telegraph message before leaving for work. Should Cassidy tell The Kid to meet him at the train or to meet him at the bank? (Assume that Cassidy is certain that The Kid will get the message before tonight, and that The Kid will believe Cassidy's stated plans.)

Since Cassidy gets to go first, we can represent this problem using the following game tree, where I've used backwards induction to indicate which action each player should take at each of his decision nodes:



No matter where Cassidy tells The Kid to meet him, The Kid will go there, because following instructions (and ending up in the same place) always gives The Kid a better payoff than ignoring instructions (and ending up in different places). So, if Cassidy tells The Kid to meet him at the train, they'll play (Train, Train). If he tells The Kid to meet him at the Bank, they'll play (Bank, Bank). Cassidy gets a higher payoff in the PSNE where they both rob the train, so he'll tell The Kid to meet him at the train.

d. Suppose instead that The Kid is the one who gets to send the message. Should The Kid tell Cassidy to meet him at the train or to meet him at the bank?

The logic here is similar to the previous part: wherever The Kid tells Cassidy to go, Cassidy will follow instructions. The Kid prefers the PSNE (Bank, Bank) over the PSNE (Train, Train), so he'll tell Cassidy to meet him at the bank.

e. Finally, suppose that Cassidy and The Kid always share their loot 50/50 after each of the robberies they commit together. If both bandits expect the agreement to be honored (i.e., enforced), how would it change your answer to part d? Who will be tempted to cheat after the robbery by refusing to share his loot? If Cassidy and The Kid plan to work together for a long time, why might this fact encourage them to honor their agreement tonight?

If Cassidy and The Kid divide their loot equally after the robbery, they would get 5 each from the train and 4 each from the bank. Since $5 > 4$, The Kid will now tell Cassidy to meet him at the train—provided he expects Cassidy to honor the agreement.

After the robbery, however, Cassidy will be tempted to escape with his payoff of 7 instead of sharing his loot with The Kid. But if they plan to work together for a long time, then Cassidy's long-term benefits from future collaboration may outweigh the short-term benefits he'd get from cheating The Kid, so Cassidy may honor the agreement out of self-interest, knowing that The Kid probably won't want to continue working with him if he gets betrayed by Cassidy.

