Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Section Problems #5
(Week of Monday, November 4)

Who's on first?

Two firms compete by choosing quantities, as in the Cournot and Stackelberg models of oligopoly. However, unlike the continuous versions we discussed in class, suppose that each firm can only choose among three levels of output (Low, Medium, High):

		Firm 2		
		Low	Medium	High
Firm 1	Low	18, 18	15, 20	9, 18
	Medium	20, 15	16, 16	8, 12
	High	18, 9	12, 8	0, 0

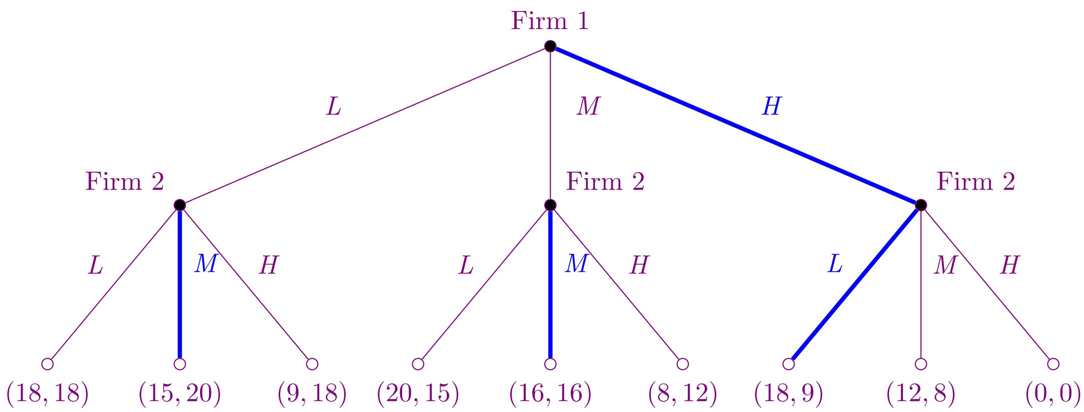
a. Suppose that the firms choose their quantities at the same time, as in the Cournot model. Find the pure strategy Nash equilibrium. What are the equilibrium payoffs?

See the circled payoffs above (I've circled Firm 1's payoffs in blue and Firm 2's in red for added clarity). The (unique) PSNE is (Medium, Medium), which is the only pair of strategies for which both players are playing a best response against the opposing strategy. The equilibrium payoffs are (16, 16).

b. Now suppose Firm 1 moves first, followed by Firm 2. (As in the dynamic games we studied in class, assume that Firm 2 observes which action Firm 1 chooses.)

Draw the game tree, then find the subgame perfect Nash equilibrium. (Be sure to specify each firm's complete "if-then" strategy.) What sequence of actions will we see the firms actually play if they use these strategies? What are the equilibrium payoffs?

Here's the game tree, with the backwards-induction solution shaded in:



The SPNE is as follows:

- Firm 1's strategy: "High".
- Firm 2's strategy: "If Firm 1 chooses Low or Medium, we will choose Medium. If Firm 1 chooses High, we will choose Low."

In this equilibrium, the sequence of actions that we'll actually see the players use are "High" by Firm 1, followed by "Low" by Firm 2. The equilibrium payoffs are (18, 9).

Intuition: as with the pharmaceutical game we studied in class (and as with the continuous version of the Stackelberg model), this game exhibits first-mover advantage: because Firm 1 moves first, it is able to commit to a High level of output, which causes Firm 2 to respond by producing a Low level of output. Holding Firm 1's action constant, the less Firm 2 produces, the bigger a payoff Firm 1 receives. So Firm 1 has an incentive to try to deter Firm 2 from producing very much.

c. Consider this pair of strategies:

- Firm 1's strategy: "Low".
- Firm 2's strategy: "If Firm 1 chooses Low, we'll choose Medium. Otherwise, we will choose High."

Is this pair of strategies a Nash equilibrium? If yes, is it subgame-perfect?

Yes, this is a Nash equilibrium! If we give the players this "script" and tell them to play these strategies—and if each player expects the other player to "stick to the script"—then neither firm can increase its payoff by deviating from the script. Let's consider each player's incentives.

First, Firm 1 doesn't want to deviate because doing so triggers a "punishment" from Firm 2: if Firm 1 produces anything other than Low, then Firm 2 will "flood the market" by picking a high level of output, and this will really hurt Firm 1's profits. (In a "real" Stackelberg problem, Firm 2's response of flooding the market would result in a low equilibrium price, which is bad for Firm 1.) If it believes that Firm 2 will really carry out this "threat", then Firm 1's best response is indeed to play "Low".

Second, if both players follow the script, then Firm 2 will end up receiving its highest possible payoff. Deviating from the script would either leave Firm 2's profits unchanged or result in Firm 2 receiving lower profits, so Firm 2 also has no reason to deviate.

Since neither player has any incentive to deviate *if it expects the other player to "stick to the script"*, these strategies form a Nash equilibrium.

However, this Nash equilibrium is not subgame-perfect because Firm 2's "threat" is not credible: if Firm 1 chooses Medium or High, then it will be in Firm 2's interest to respond by playing Medium or Low (respectively), regardless of what Firm 2 *said* it was going to do. Since both players are rational, Firm 1 knows that Firm 2 is bluffing: Firm 1 simply ignores the bluff and chooses to play High, and then Firm 2 responds by playing Low, since doing so gives Firm 2 its best payoff under the circumstances.

Fortune favors the bold

Consider a duopoly market in which total demand is given by $p(Q) = 72 - 2Q$, where $Q = q_1 + q_2$. The firms have identical costs, given by $C_1(q_1) = 24q_1$ and $C_2(q_2) = 24q_2$.

a. Suppose the firms choose their quantities at the same time (as in the Cournot model). Find the Nash equilibrium quantities q_1^* and q_2^*. Compute each firm's profits.

We start by solving for each firm's best-response function. For Firm 1, the profit-maximization problem is

$$\max_{q_1} \ \pi = (72 - 2q_1 - 2q_2)q_1 - 24q_1$$

Taking the FOC and rearranging gives us

$$q_1^* = BR_1(\hat{q}_2) = 12 - \frac{1}{2}\hat{q}_2$$

Since the firms are identical, we can use symmetry to calculate Firm 2's best-response function as

$$q_2^* = BR_2(\hat{q}_1) = 12 - \frac{1}{2}\hat{q}_1$$

Combining these two FOCs and solving the system of two equations, we find

$$q_1^* = 8 \qquad \text{and} \qquad q_2^* = 8$$

Since the firms are identical in all respects (including the fact that they move the same time), it makes sense that they would each produce the same amount. So this is a reassuring "check" that we did the math right. Given these quantities, the price is

$$p^* = 72 - 2(8 + 8) = 40$$

which implies that Firm 1 makes a profit of

$$\pi_1 = p^* q_1^* - C_1(q_1^*) = (40)(8) - (24)(8) = 128$$

Once again, symmetry implies that Firm 2 makes the same profit of 128. (We can also calculate Firm 2's profits directly using p^* and q_2^* and come to the same conclusion.)

b. Now suppose that Firm 1 chooses its quantity first, followed by Firm 2 (as in the Stackelberg model). Find the Nash equilibrium quantities q_1^* and q_2^*. Show that, relative to the Nash equilibrium we saw under Cournot, Firm 1's profits have gone up and Firm 2's profits have gone down.

We solve the Stackelberg problem by working backwards, starting with Firm 2 and then working back to Firm 1. To start, we can recycle Firm 2's best-response function from the previous part:

$$q_2^* = BR_2(\hat{q}_1) = 12 - \frac{1}{2}\hat{q}_1$$

Next, Firm 1 maximizes its profits (after substituting Firm 2's best-response function into the profit function):

$$\max_{q_1} \ \pi = \left(72 - 2q_1 - 2\left(12 - \frac{1}{2}q_1\right)\right) q_1 - 24q_1 \implies \max_{q_1} \ \pi = (24 - q_1)\, q_1$$

Taking the FOC and solving for q_1 yields $q_1^* = 12$. Plugging this back into Firm 2's best-response function, we find $q_2^* = 6$. These quantities imply that p has fallen to

$$p^* = 72 - 2(12 + 6) = 36$$

Calculating each firm's profits, we find that Firm 1 makes a profit of 144 (greater than its Cournot profit of 128) whereas Firm 2 makes a profit of 72 (less than its Cournot profit of 128). In class, I made a "revealed preference" argument that the ability to move first must increase Firm 1's profits (since it could have chosen the Cournot quantity, but didn't). The calculation here confirms this intuition directly.