Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Section Problems #6
(Week of Monday, November 18)

Perc

Traditional dry-cleaning methods use a chemical called "perc" that's believed to increase the risk of cancer. In this problem, we'll see how the negative externality caused by perc pollution affects the socially optimal amount of dry cleaning.

Suppose that Sacramento has a perfectly competitive dry cleaning industry, with a market-level demand curve given by

$$p(Q) = 50 - 5Q$$

Suppose that each dry cleaner has (private) costs of production given by

$$PC(q) = 20q$$

(Remember that the private marginal cost curve equals the supply curve.)

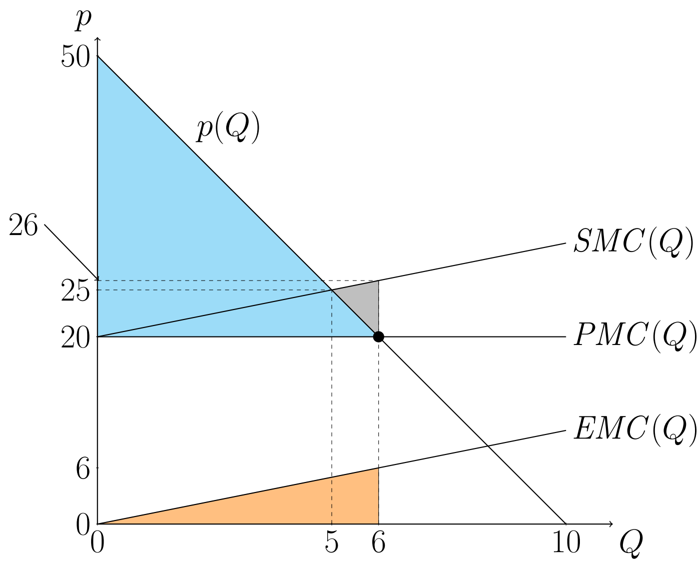
Finally, suppose that dry cleaning imposes external costs given by

$$EC(Q) = \frac{1}{2}Q^2$$

Notice that the external marginal cost, $EMC(Q) = \frac{d}{dQ}\left(\frac{1}{2}Q^2\right) = Q$, is an increasing function of total output (Q), reflecting the idea that small levels of perc pollution are not very harmful but increased exposure becomes more and more dangerous at higher levels of exposure.

a. Plot the demand curve, private marginal cost curve, external marginal cost curve, and social marginal cost curve.

   See below. Looking ahead to the next few parts of this problem, I've indicated the competitive equilibrium outcomes (Q_c, p_c), and I've shaded in the consumer surplus (blue), external cost (orange), and deadweight loss (gray) in the competitive equilibrium.



b. Compute the (unregulated) competitive equilibrium quantity Q_c and price p_c. What is the *consumer* surplus in this equilibrium? What is the *total* surplus?

In the competitive equilibrium, supply equals demand: $p(Q) = PMC(Q)$. We can also say that the private marginal benefit is set equal to the private marginal cost:

$$50 - 5Q = 20 \implies Q_c = 6 \implies p_c = 50 - 5(6) = 20$$

The consumer surplus is the area under the demand curve and above the price. The area of this triangle is

$$CS = \frac{1}{2}(6-0)(50-20) = 90$$

Since there are no taxes, subsidies, or external benefits here, the total surplus is $TS = CS + PS - EC$. The producer surplus is 0. The external cost is $EC(Q_c) = \frac{1}{2}Q_c^2 = \frac{1}{2}6^2 = 18$. Therefore, the total surplus is $TS = 90 - 18 = 72$.

c. Now compute the socially optimal quantity Q_s. Compute total surplus using the formula $TS = SB - SC$ (total surplus equals social benefits minus social costs).

We find the socially optimal quantity by setting the social marginal benefit equal to the social marginal cost. Since there are no external benefits in this problem, the *social* marginal benefit is just equal to the *private* marginal benefit—in other words, the demand curve:

$$50 - 5Q = 20 + Q \implies Q_s = 5$$

The social benefits are the area under the social marginal benefit curve (which, again, is just the private marginal benefit curve in this case because there are no external benefits). This area consists of a rectangle plus a triangle:

$$SB = 5 \times 25 + \frac{1}{2} \times 5 \times (50 - 25) = 187.5$$

The social costs are the area under the social marginal cost curve. This is also a rectangle plus a triangle:

$$SC = 5 \times 20 + \frac{1}{2} \times 5 \times (25 - 20) = 112.5$$

Taking the difference, we find that $TS = 187.5 - 112.5 = 75$.

d. Compute DWL in the competitive equilibrium. How does it compare to the difference in total surplus between the competitive equilibrium and the social optimum?

The deadweight loss is the area shaded gray in the graph above:

$$DWL = \frac{1}{2}(6 - 5)(26 - 20) = 3$$

The deadweight loss is exactly equal to the difference in total surplus between the competitive equilibrium and the social optimum. (Remember: the deadweight loss can be *defined* as being equal to this difference.)

e. Suppose that the government imposes a corrective tax equal to \$5 per unit produced. Find the new competitive equilibrium quantity Q_c^{tax}. Then find the total surplus.

The tax increases the private marginal cost of production to $PMC(Q)+t = 20+5 = 25$. In other words, it shifts the supply curve up by \$5. We find the new competitive equilibrium quantity by setting the demand curve equal to this new supply curve. They intersect where

$$50 - 5Q = 25 \implies Q_c^{\text{new}} = 5$$

Notice that this corrective tax has gotten us to the social optimum. (It's the optimal corrective tax.)

Mending Wall

. . . I let my neighbour know beyond the hill;
And on a day we meet to walk the line
And set the wall between us once again.
We keep the wall between us as we go.
To each the boulders that have fallen to each.
And some are loaves and some so nearly balls
We have to use a spell to make them balance:
"Stay where you are until our backs are turned!"
We wear our fingers rough with handling them.
Oh, just another kind of out-door game,
One on a side . . .
—Robert Frost, "Mending Wall"

Two neighbors, Alessandra and Elinor, share a fence between their property. The fence is in poor condition and each neighbor is deciding how many units of fencing to replace.

Let $Q = q_A + q_E$ denote the total number of renovated units of fencing, where q_A and q_E are the quantities contributed by each neighbor. Alessandra's private marginal benefit from fencing (i.e., her demand curve) is given by

$$p_A(Q) = \begin{cases} 90 - Q & \text{for } Q \leq 90 \\ 0 & \text{for } Q > 90 \end{cases}$$

Elinor's private marginal benefit is given by

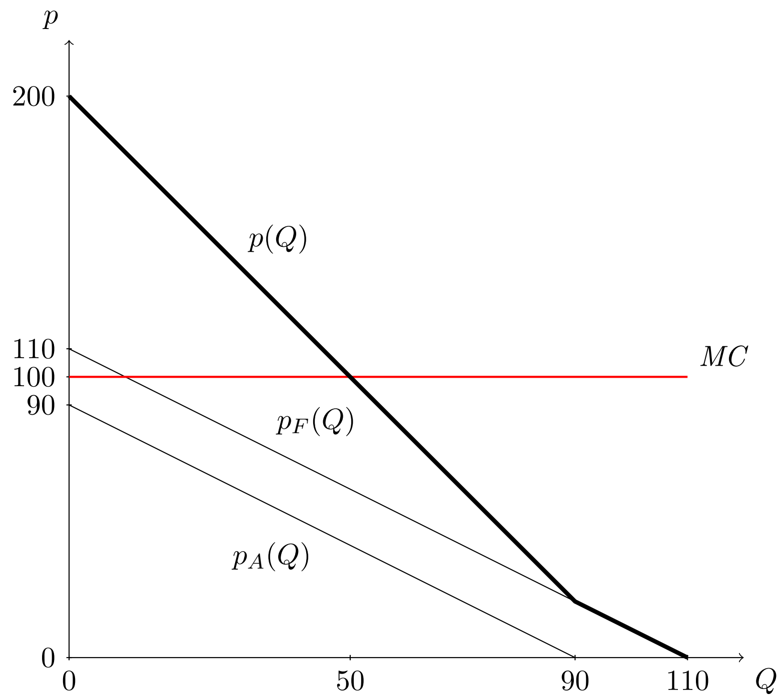
$$p_E(Q) = \begin{cases} 110 - Q & \text{for } Q \leq 110 \\ 0 & \text{for } Q > 110 \end{cases}$$

a. Plot the individual and social demand (i.e., marginal benefit) curves.

The graph is below. (I've included the marginal cost introduced in the next part.) The individual demand curves are given above and the social demand curve is

$$p(Q) = \begin{cases} 200 - 2Q & \text{for } Q \leq 90 \\ 110 - Q & \text{for } Q > 90, Q \leq 110 \\ 0 & \text{for } Q > 110 \end{cases}$$

which we obtain by vertically summing the two individual demand curves.



b. Suppose that each unit of fencing costs \$100. What is the socially optimal number of units of fencing (Q_s)? If Alessandra and Elinor play a static game, how much fencing will each neighbor purchase (i.e., what are the Nash equilibrium quantities q_A^* and q_E^*)?

The socially optimum is $Q_s = 50$, which we obtain by finding the point where the social demand curve intersects the supply (marginal cost) curve. This intersection occurs on the first segment of the social demand curve:

$$p(Q) = 200 - 2Q = 100 \implies Q_s = 50$$

In the static game, since Alessandra's marginal benefit is never more than 90, she is never willing to pay for fencing herself, so $q_A^* = 0$. (In other words, this is a strictly dominant strategy for Alessandra.) Knowing that Alessandra will do nothing, Elinor buys fencing up to the point where her private marginal benefit equals the marginal cost: $110 - q_E^* = 100 \implies q_E^* = 10$. The total quantity supplied is $q_A^* + q_E^* = 10$, which is less than the social optimum $Q_s = 50$.

c. Now suppose that each unit of fencing costs \$150. What is the socially optimal number of units of fencing (Q_s)? How much fencing will each neighbor choose to purchase?

The social optimum is now $Q_s = 25$, which we compute the same way as above: $200 - 2Q = 150 \implies Q_s = 25$. Now, however, neither neighbor buys any fencing since their private marginal benefit is always lower than the marginal cost: $q_A^* = q_E^* = 0$.