Intermediate Microeconomic Theory
ECN 100B, Fall 2019
Professor Brendan Price

Section Problems #8
(Week of Monday, December 2)

## I could've sworn I parked here . . .

Consider a Davis student whose utility function is $u(w) = \sqrt{w}$, where $w \geq 0$ is her wealth, which equals her financial wealth $w_f$ plus the value of her bike $w_b$:

$$w = w_f + w_b$$

The student starts off with \$144 in cash ($w_f = 144$) and owns a bike worth \$81 ($w_b = 81$). There is a 1/3 chance the student's bike is stolen. Otherwise, it's fine.

a. Is this student risk-averse, risk-neutral, or risk-loving? Prove this mathematically.

b. Compute each of the following:

   (i) The expected loss from bike theft (i.e., the expected value of the wealth she loses).

   (ii) The student's expected utility.

   (iii) The student's certainty equivalent. (An agent is indifferent between playing a "lottery" and getting the corresponding certainty equivalent for sure.)

   (iv) The student's risk premium. (The risk premium is the amount she'd be willing to give up to get her expected wealth for sure. It equals expected wealth minus the certainty equivalent.)

c. Kryptonite's "New York" bike lock is supposed to keep your bike safe even on the mean streets of Manhattan. Suppose that purchasing the New York lock completely eliminates the risk of bike theft.

   (i) How much would this student be willing to pay for the New York lock?

   (ii) Now suppose that the student has just won a Powerball lottery, increasing her financial wealth to $w_f = 1600$. Compute her new willingness to pay for the lock.

   (iii) The results above suggest that richer students may have *lower* willingness to pay for a bike lock. Can you think of a reason why richer students might have *higher* willingness to pay? (I can think of two main reasons.)

# Playing the stocks

There are two stocks, $A$ and $B$. One share of either stock is worth \$30 with probability $1/2$ and it's worth \$50 with probability $1/2$.

a. Compute the expected value and variance of a share of stock $A$.

b. Compute the expected value of a portfolio consisting of one share of $A$ and one of $B$.

c. Compute the variance of this portfolio under each of the following assumptions:

   (i) The shares are perfectly negatively correlated: whenever stock $A$ performs well (equaling \$50), stock $B$ performs badly (equaling \$30), and vice versa.

   (ii) The shares are perfectly positively correlated: whenever stock $A$ performs well, so does stock $B$; whenever stock $A$ performs badly, so does stock $B$.

   (iii) The shares are uncorrelated: their values are independent of one another.

d. Under which of these assumptions does diversification (buying one share of each stock) reduce the portfolio's variance, relative to buying two shares of the same stock?