Graduate Labor Economics

Notes to Accompany Lecture 12
Joblessness and Job Search

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Job search is a leading paradigm for thinking about labor markets: it underlies a rich literature in applied microeconomics and is also the main point of contact between labor economics and macroeconomics. This note introduces the basic theory of job search (see, e.g., Mortensen, 1977). We'll focus here on the determinants of job search at the individual level, abstracting from many possible generalizations as well as from (interesting) general equilibrium considerations like how one worker's choice of search effort affects other workers' probability of finding a job. I'll develop the theory in continuous time, focusing on the simpler case of a stationary environment.[1]

1 Model setup

- A worker loses her job at date 0 and searches for work until reemployed. We want to characterize her search behavior and to see how it responds to changes in the economic environment.
- While unemployed, the worker receives an unemployment insurance (UI) benefit equal to b. Since we're focusing on the stationary case, we'll assume these benefits are paid in perpetuity.
- By paying a search cost $\psi(s)$, she can generate a flow of job offers that arrive at continuous rate $s \geq 0$. We'll assume that $\psi(\cdot)$ is increasing, convex, and satisfies the Inada conditions $\psi(0) = 0$, $\psi'(0) = 0$, and $\lim_{s\to\infty} \psi'(s) = \infty$, which will ensure that the worker always chooses an interior optimum for her search intensity.
- Each job offer is drawn from a continuous, stationary offer distribution $G(w)$. We'll assume that $G(b) < 1$, so that searching for a job is potentially worthwhile. The worker knows $G(\cdot)$ from the beginning: there is no learning in this model.
- Once the worker accepts a job offer, she remains employed at that wage forever (it's straightforward to allow for exogenous job separations). If she rejects an offer, it is gone for good.[2]

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[1] It's good to be comfortable working in either discrete or continuous time. Discrete time is sometimes easier to think about, but continuous time is more tractable in the presence of competing risks. For example, suppose that workers can become reemployed either by finding a new job or by being recalled from temporary layoff (Katz, 1986). In discrete time, we have to deal with the pesky possibility that a new-job offer and a recall offer arrive simultaneously. In continuous time, the probability of both events coinciding is zero, so we don't have to worry about this issue.

[2] This use-it-or-lose-it assumption is innocuous in the stationary case because a job that is rejected today will never be deemed acceptable in the future. With non-stationarity, however, a worker might want to reconsider jobs she previously rejected. See Boone and van Ours (2012) for a model with "storable" job offers, which can rationalize why job-finding tends to "spike" rather than "plateau" when UI benefits are exhausted (Meyer, 1990).

- To simplify the notation, suppose that the worker is risk-neutral. (This is easy to relax.) Then she maximizes the expected present discounted value of consumption, net of the costs of job search. She discounts the future at continuous rate $\delta > 0$. For simplicity, we'll assume she cannot borrow or save. Again, this can be relaxed, but if we allow for savings we'll introduce both another choice variable and another state variable, so the math will get harder.

2 Search strategies

- In this environment, a strategy is a complete contingent plan specifying the worker's (i) choice of search effort and (ii) decisions about whether to accept/reject a given job offer, given the entire sequence of past actions and past offer realizations.
- In principle there is a very large strategy space here, but the optimal strategy won't depend on past behavior or past realizations: workers are "memoryless", since past realizations convey no information and are irrelevant to future utility flows. So, in practice, we're looking for a sequence of search efforts $s_t^* \in [0, \infty)$ and a sequence of acceptance functions $a_t^*(w) \in \{0, 1\}$ indicating whether a worker accepts/rejects a job offering w at time t.
- Under the assumptions we've made, this is a stationary problem, so if s_t^* is optimal at time t, it is optimal at all time periods; likewise, the same acceptance function applies at every point in time. So we're looking for stationary strategies s^* and $a^*(w)$.

3 The Bellman equation and the reservation wage

- The model described above implies the Bellman equation

$$\delta U = \max_s \ b - \psi(s) + s \int_0^\infty \max\{0, J(w) - U\} dG(w) \tag{1}$$

where U is the value of unemployment and $J(w)$ is the value of being employed at wage w.[3] Note that the integrand $\max\{0, J(w) - U\}$ reflects the fact that the worker will accept a job offering w if and only if that job yields higher expected utility than remaining unemployed.

- The lefthand side δU is analogous to the "capital cost" associated with the "asset" of being unemployed. At the optimum, this capital cost must exactly equal the asset return, which is the flow value of unemployment $(b - \psi(s^*))$—under the asset analogy, this is the "dividend"—plus the upside risk of potentially exchanging unemployment for a more valuable asset $J(w)$. Bellman equations usually also have a "capital gain" term capturing changes in the (asset) value of remaining unemployed, but since we're in a stationary world, that term is zero.
- So far we haven't said anything about the acceptance function $a^*(w)$. But clearly, if it's optimal for a worker to accept a job offering w, then it's also optimal to accept any job offering $w' > w$. This leads us to a cutoff rule: the worker chooses a *reservation wage* $\underline{w}$ such that $a^*(w) = \mathbb{1}\{w \geq \underline{w}\}$: jobs are accepted if and only if they offer more than the reservation wage. This allows us to drop the more general $a(\cdot)$ notation and express the worker's strategy in terms of s^* and $\underline{w}^*$.

[3] One approach to deriving the Bellman equation is to start with the discrete case (which you may find easier to think about) and then develop the continuous case by analogy, or by taking limits as time periods get arbitrarily small.

- What pins down the reservation wage? Well, by definition, the worker should be indifferent between accepting or rejecting a job that offers $\underline{w}$. This implies that

$$J(\underline{w}) = U \tag{2}$$

- We can now rewrite the Bellman equation. For $w < \underline{w}$, $J(w) - U = J(w) - J(\underline{w}) < 0$, so the integrand $\max\{0, J(w) - U\}$ evaluates to zero. That means we have

$$\delta U = \max_s \; b - \psi(s) + s \int_{\underline{w}}^{\infty} (J(w) - U) dG(w) \tag{3}$$

- With a bit of calculus, we can further rewrite the Bellman as

$$\delta U = \max_s \; b - \psi(s) + s(1 - G(\underline{w}))(\mathbb{E}(J(w) \mid w \geq \underline{w}) - U) \tag{4}$$

  where $1 - G(\underline{w}) = \Pr\{w \geq \underline{w}\}$ is the probability of drawing an acceptable wage offer.

- If the worker accepts a job at wage w, then she obtains lifetime utility $\int_0^\infty e^{-rt} w \; dt = \frac{w}{\delta}$, which means that $J(w) = \frac{w}{\delta}$. (This also establishes that $J(w)$ is increasing in w, which I implicitly used above.)

- The flow rate of exits from unemployment (conditional on still being at risk) is $s(1 - G(\underline{w}))$. This is the instantaneous *hazard rate* of finding a job, which I'll denote $\lambda \equiv s(1 - G(\underline{w}))$.

- A useful check on the meaning of U is to consider what happens if the worker simply sets $s^* = 0$ and stays out of work forever. If that were optimal, we'd have $\delta U = b \implies U = \frac{b}{\delta}$, which is the lifetime utility associated with simply consuming b every period and never searching. Under the assumptions we've made, the worker can do strictly better by searching a positive amount, but the flow of UI benefits puts a lower bound on lifetime utility.[4]

- It's also instructive to rewrite the Bellman as follows:

$$\delta U = b - \psi(s^*) + s^*(1 - G(\underline{w}))(\mathbb{E}(J(w) \mid w \geq \underline{w}) - U) \tag{5}$$

  or equivalently

$$U = \frac{(b - \psi(s^*) + s^*(1 - G(\underline{w}))\mathbb{E}(J(w) \mid w \geq \underline{w})}{\delta + s^*(1 - G(\underline{w}))} \tag{6}$$

- Suppose that the government increases the generosity of UI benefits ($b \uparrow$). We can quantify the welfare impact of this change by invoking the envelope theorem, which lets us treat s^* as a constant when computing the marginal change in the value of unemployment:

$$\frac{dU}{db} = \frac{1}{\delta + s^*(1 - G(\underline{w}))} > 0 \tag{7}$$

  The welfare impact is discounted at the effective rate $\delta + s^*(1 - G(\underline{w})) = \delta + \lambda^*$: the sooner the worker expects to escape from unemployment, the less a change in UI benefit generosity affects her welfare.

[4] Note that $U > \frac{b}{\delta}$, which implies that $\underline{w} > b$: workers will reject some jobs that offer $w > b$ because, upon accepting a job, they relinquish the option value of staying unemployed and possibly finding an even better job. (If we assume that workers can engage in on-the-job search, and if the cost of search is the same regardless of employment status, we instead find that $\underline{w} = b$: workers accept any job that offers higher flow utility than staying unemployed does.)

4 Optimal search intensity and comparative statics

- Next, we want to characterize the worker's choice of search intensity, s^*, and see how it responds to changes in economic conditions. Start by taking the FOC with respect to s:

$$\psi'(s^*) = (1 - G(\underline{w}))(\mathbb{E}(J(w) \mid w \geq \underline{w}) - U) \tag{8}$$

  The LHS is the marginal cost of search, and the RHS is the marginal benefit. The worker searches until these are exactly equal.

- How does an increase in b affect search effort, the reservation wage, and the hazard rate of reemployment? For the reservation wage condition, we can simply differentiate Equation (2):

$$\frac{\underline{w}}{\delta} = U \implies \frac{d\underline{w}}{db} = \delta \frac{dU}{db} > 0 \tag{9}$$

  Generous benefits beget picky workers.

- For the search-intensity condition, apply the implicit function theorem to Equation (8):

$$\frac{ds^*}{db} = -\frac{(1 - G(\underline{w}))}{\psi''(s^*)} \frac{dU}{db} < 0 \tag{10}$$

  When benefits are high, workers don't search as hard.

- Both of these effects point towards a lower hazard rate, so that $\frac{d\lambda^*}{db} < 0$. A vast empirical literature (e.g., Meyer, 1990; Katz and Meyer, 1990) has confirmed the theoretical result that increases in UI generosity lead to longer jobless spells (though such increases may still be welfare-improving).

- Also note that, in this model, generous UI benefits lead to higher wages: when reservation wages rise, workers end up accepting higher-paying jobs on average. This is a popular justification for UI: that it helps workers find "good jobs" rather than accepting the first offer that comes along. In more general models, however, UI can potentially *reduce* reemployment wages by incentivizing workers to take longer jobless spells that lower their earnings capacity (Schmieder et al., 2016; Nekoei and Weber, 2017; Price, 2019). The empirical literature has tended to find small effects, with the sign varying across studies.

5 (A few of the many) extensions

- A large literature dating back to Baily (1978) characterizes *optimal unemployment insurance*. The key question is how to balance the moral hazard costs of weakening job-search incentives against the consumption smoothing benefits of insuring workers who get laid off.
  - For a constant benefit level (that lasts indefinitely), the Baily-Chetty formula can be written as

$$\gamma \frac{\Delta c}{c}\bigg|_{b^*} = \varepsilon_{D,b} \tag{11}$$

    where the lefthand side is the consumption drop during unemployment ($\frac{\Delta c}{c}$) times the coefficient of relative risk aversion (γ, capturing the curvature of the utility function) and the righthand side is the elasticity of jobless durations with respect to the benefit level. An optimal UI policy sets the marginal (consumption smoothing) benefits equal to the marginal (moral hazard) cost.

  - Chetty (2006) substantially generalizes Baily's original result through heavy use of the envelope theorem: for a wide class of models, the costs and benefits of a marginal change in UI generosity can be expressed in terms of the elasticity of jobless durations with respect to UI, the consumption drops incurred by job-losers, and a calibrated value for the coefficient of relative risk aversion.
  - Recent work has pushed the optimality debate beyond the original framework of a constant UI benefit level to richer questions about how to structure short- vs. long-term UI benefits (Kolsrud et al., 2018) and whether UI generosity should vary with the business cycle (Schmieder et al., 2012; Kroft and Notowidigdo, 2016).

- Chetty (2008) points out that UI affects job search both by distorting work incentives and by providing temporary liquidity. He offers empirical evidence (based on cross-sectional heterogeneity in household assets among job-losers, as well as variation in severance pay among job-losers) that UI has a much smaller impact on jobless durations when workers have sufficient liquidity to smooth over a job loss—suggesting that much of the "moral hazard cost" of UI is really a beneficiai easing of credit constraints.

- Schmieder et al. (2016) allow for changes in the job-offer distribution as a function of jobless duration. Workers may receive worse offers the longer they are unemployed, e.g. because of statistical discrimination by employers (i.e., "scarring": see Kroft et al., 2013) or skill depreciation that accrues with time out of work.

- Katz (1986), Katz and Meyer (1990), and Nekoei and Weber (2015) distinguish between new jobs and recalls from temporary layoff. Recalls are historically quite common and account for a sizable share of new flows into unemployment, and the recall hazard exhibits a very different time pattern (initially rising with jobless duration, peaking after a few months, and then declining) from the new-job hazard (which declines more uniformly with duration).

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